



Differential Evolution and Genetic Algorithm for Sidelobe Reduction of A Concentric Ring Array Antenna by Radial Variation of Amplitudes With Fixed and Variable First Null Beamwidth

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Abstract: Reduction of side lobe levels in a concentric ring arrays results in wide first null beamwidth. The authors propose a pattern synthesis method based on the Differential Evolution (DE) algorithm to reduce the side lobe levels of a concentric ring array of isotropic antennas while keeping the first null beamwidth (FNBW) fixed and variable by radial variation of amplitudes of the array elements. Two different cases have been studied, one with fixed initial interelement distance and another with optimum interelement distance for the entire array. The FNBW of the optimized array is kept equal to or less than that of a uniformly excited and 0.5λ spaced concentric ring array of the same number of elements and rings. Results are also compared with Genetic Algorithm to establish its superiority.

Keywords: Circular ring array, Differential Evolution (DE) algorithm, first null beam width (FNBW), optimization, side lobe level, Genetic algorithm.

1. Introduction

A circular ring array, also known as a concentric circular array (CCA) is a planar array that consists of one or more concentric rings, each having equally spaced array elements on its circumference. An important property of a CCA is that, its beam pattern remains invariant for 360° azimuthal coverage if the array consists of several rings with an appreciably large number of elements in each ring. Its main attraction is the cylindrical symmetry of its radiation pattern and compact structure. One of the important configurations regarding CCA is the uniform concentric circular array (UCCA) where the inter-element spacing in each individual ring is kept almost half of the wavelength and all the elements in the array are uniformly excited. However, in its modest form the array suffers from a high side lobe problem. Generally low side lobes in the array factor are obtained through optimum amplitude weights of the signals at each array element.

The radiation pattern function of a concentric ring array has been expressed by Stearns and Stewart [1] as a truncated Fourier-Bessel series and the non uniform distribution of the rings has been approximated to a smaller number of equally spaced ones. N. Goto and D. K Cheng showed that for a Taylor weighted ring array the maximum allowable inter-element spacing should be about four-tenths of a wavelength, if high side lobes are to be avoided [2]. L. Biller and G. Friedman used steepest descent iterative process to find out element weights and ring spacing to get lower sidelobe levels and control over beam width [3]. D. Huebner reduced the sidelobe levels for small concentric ring array by adjusting the ring radii using optimization technique [4]. B. P. Kumar and G. R. Branner also proposed optimum ring radii for getting lower sidelobes [5]. M. Dessouky, H. Sharshar and Y. Albagory showed that the existence of central element in concentric circular array of smaller innermost ring reduced the sidelobe levels significantly while minor increase in the beamwidth [6]. Sidelobe levels can be reduced by thinning the array [7-8]. The array is thinned by turning off selected elements from the

where, M = Number of concentric rings, N_m = Number of isotropic elements in m-th ring, I_m = excitation amplitude of elements on m-th circular ring, d_m = inter-element arc spacing of m-th circle, $r_m = N_m d_m / 2\pi$ is the radius of the mth ring, $\phi_{mn} = 2n\pi / N_m$ is the angular position of mn-th element with $1 \leq n \leq N_m$, θ, φ = polar and azimuth angle, λ = wave length, k = wave number = $2\pi/\lambda$, j = complex number, ϕ_m = excitation phase of elements on m-th ring, All the elements have same excitation phase of zero degree.

Side lobe levels of a uniform concentric ring array can be reduced by finding out a suitable set of radial amplitude distribution of the array elements, which is based on the assumption that all the array elements on the same circle have same amplitude distribution, but they vary from ring to ring.

The number of elements in m-th ring of a concentric ring array can be expressed as:

$$N_m = \frac{2\pi r_m}{d_m} \quad (3)$$

In this problem, two different cases have been considered. For the first case, the interelement distance d_m for the entire array is kept at 0.5λ . Then an optimum set of radial amplitude distribution for the entire array is determined using DE such that the optimized array gives lower side lobe levels while retaining other desire array characteristics.

In the second case, optimum values of radial amplitudes and optimum interelement distance d_m for the entire array are determined using DE algorithm to get lower sidelobe levels with desire array characteristics. In the second case d_m is varied in such a way that it lies between $0.5\lambda \leq d_m \leq \lambda$.

The number of elements in each ring is determined using equation (3).

Since the number of elements in a particular ring must be an integer quantity, so only the computed integer values of equation (3) are taken

The fitness functions for this problem are given by:

$$Fitness1 = k_1 \max SLL + k_2 (FNBW_o - FNBW_d)^2 H(T) \quad (4)$$

$$Fitness2 = \max SLL \quad (5)$$

Where $\max SLL$ is the value of maximum sidelobe level, $FNBW_o$, $FNBW_d$ are the obtained and desired values of first null beam width respectively, k_1 , k_2 are weighting coefficients to control the relative importance given to each term of equation (4) and the values are chosen as, $k_1=1$ and $k_2=100$ respectively. $H(T)$ is Heaviside step functions defined as:

$$T = (FNBW_o - FNBW_d) \quad (6)$$

$$H(T) = \begin{cases} 0, & \text{if } T < 0, \\ 1, & \text{if } T \geq 0 \end{cases} \quad (7)$$

Equation (5) is for not keeping FNBW fixed. Equation (4) and equation (5) are minimized using DE for optimal synthesis of array.

3. Differential Evolution Algorithm

Differential Evolution is a simple evolutionary algorithm introduced by Storn and Price [11]. Similar to GA [17, 19-20], DE is also an algorithm based on population. DE algorithm is a stochastic optimization method for minimizing an objective function that can model the problem's objectives while incorporating constraints. The algorithm mainly has three

advantages: ability to find the true global minima regardless of the initial parameter value, converges fast and uses a few control parameter [11-14]. DE first samples the objective function at multiple, randomly chosen initial points. Then NP (Number of populations) vectors in the initial population are chosen from the predefined parameter bounds. To explore the objective functions landscape, DE employs a difference between the parameter vectors. New points (trial solution) are generated through perturbations of existing points. DE perturbs vectors with the scale difference of two randomly selected population vectors. In the next stage (selection) the trial vector competes against the population vector of the same index. Once the last trial vector has been tested, the survivors of the NP pair wise competitions become parents for the next generation in the evolutionary cycle. The algorithm can be summarized as below [11-14]:

Step 1: Initialization:

The generation number is set to $t=0$ and a population of NP individuals are randomly initialized in the D -dimensional search space as:

$$P_t = \left\{ \vec{X}_1(t), \dots, \vec{X}_{NP}(t) \right\}, \text{ where } \vec{X}_i(t) = [x_{i,1}(t), x_{i,2}(t), \dots, x_{i,D}(t)] \text{ and}$$

each individuals are uniformly distributed in domain $[\vec{X}_{min}, \vec{X}_{max}]$.

Step 2: Evaluate the fitness:

Evaluate the fitness of each individual at current generation.

Step 3: Mutation:

Create donor vector $\vec{V}_i(t)$ corresponding to the i -th target vector $\vec{X}_i(t)$ for all the individuals at current generation using any one of the DE mutation scheme [10-16].

In this problem the mutation strategy known as DE/best/1 has been used and is expressed as:

$$\vec{V}_i(t) = \vec{X}_{best}(t) + F \cdot (\vec{X}_{r_1}(t) - \vec{X}_{r_2}(t)) \quad \text{for } i=1,2,\dots, NP$$

where, \vec{X}_{best} is the best vector of the current population, \vec{X}_{r_1} and \vec{X}_{r_2} are randomly picked up vectors from the current generations, F is the scale factor, $F \in (0, 1+)$, a positive real number that controls the rate at which the population evolves.

Step 4: Crossover:

Use any one of the crossover scheme in DE [14-16] to form the trial vector $\vec{U}_i(t)$, by exchanging the components of the donor vector $\vec{V}_i(t)$ and the target vector $\vec{X}_i(t)$ with a crossover probability of C_r ($C_r \in [0, 1]$), for all the individuals at current generation.

Step 5: Selection:

Select the best individuals for the next generation as follows:

$$\vec{X}_i(t+1) = \begin{cases} \vec{U}_i(t), & \text{if } f(\vec{U}_i(t)) \leq f(\vec{X}_i(t)) \\ \vec{X}_i(t), & \text{if } f(\vec{U}_i(t)) > f(\vec{X}_i(t)) \end{cases}, \quad \text{for } i=1,2,\dots, NP$$

Compute $\vec{X}_{Gbest}(t)$ at current generation as follows:

Find out the corresponding vector among NP individuals for which $f(\vec{X}_i(t+1))$, for $i=1,2,\dots, NP$, becomes minimum (for minimization problem) and assign that vector to $\vec{X}_{Gbest}(t)$ where, $f(\vec{X})$ is the function to be minimized. Since the

selection process employs a binary decision the population size remains fixed throughout generations.

Step 6:

Increase the iteration count $t = t+1$ and repeat step 2-5 until the termination condition is satisfied. Return X_{Gbest} as the result.

The termination condition can be defined:

- (i). When a fixed number of iteration for t_{max} , with a suitably large value of t_{max} , depending upon the complexity of the objective function, is reached.
- (ii). When best fitness of the population does not change appreciably over successive iterations.

Mutation demarcates one DE scheme from another. Each mutation strategy combines with either ‘exponential’ or ‘binomial’ type crossover and produce new working strategy. There are in total ten different working strategies of DE as suggested by Storn and Price [11-14].

In this problem the *DE/best/1/exp* strategy has been used along with number of population (NP) =40 and crossover rate (CR) = 0.7 and the termination condition has been defined as t_{max} =800. Number of variables for the first case is = 9 and for the second case is=10. For the first case, the lower limits for nine variables have been taken as:

$X_{min} = [0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1]$ and the upper limits are taken as:

$X_{max} = [1, 1, 1, 1, 1, 1, 1, 1, 1]$. For the second case, the lower limits for ten variables are taken as: $X_{min} = [0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.51]$ and the upper limits are taken as:

$X_{max} = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$. In the second case the lower limit of the tenth variable is taken 0.51, because the optimum value of the interelement spacing for the entire array should be more than 0.5λ in order to avoid the effect of mutual coupling.

4. Simulation Results

For a nine ring concentric ring array of isotropic antennas [7], the initial radius of the rings are $r_m = m\lambda/2$ (m-th ring) and the interelement spacing in each ring is taken as $\lambda/2$. For this arrangement the total number of isotropic elements is 279. Uniform excitation and constant phase angle between the elements gives side lobe level -17.4 dB [7] and FNBW 14.8 degree. The reason behind choosing concentric ring array is its φ -symmetric beam pattern and compact structure. In this problem, the sidelobe level of the array has been reduced based on finding out an optimum set of amplitude distribution of the array under two different cases. The obtained results using DE for both the cases are also compared with Genetic Algorithm (GA). The fitness functions for the GA are taken same as DE. The numbers of population in case of GA are taken same as that of DE. Two-point crossover along with uniform mutation of rate 0.01 and ranking selection are used. Crossover fraction is taken to be 0.07. The initial values and the termination conditions are also kept same as DE.

Case I:

In this case, interelement distance is kept fixed ($d_m = 0.5\lambda$) for the entire array. Total number of isotropic elements in the array is 279. To get lower side lobe level with FNBW below or equal to that of a nine ring uniform concentric ring array, an optimum set of radial amplitude distribution has been found out using Differential Evolution (DE) algorithm. In this way the side lobe levels has been reduced below -22 dB with fixed FNBW and below -43 dB without fixing FNBW. The obtained values of side lobe level using GA are -21.22 dB for fixed FNBW and -39.82 dB for variable FNBW.

Case II:

In this case interelement distance d_m is not prefixed but also optimized. d_m is varied in such a way that it lies between $0.5\lambda \leq d_m \leq \lambda$.

To obtain lower side lobe level with FNBW below or equal to that of a nine ring uniform concentric ring array, optimum set of radial amplitude distribution and optimum value of interelement spacing for entire array has been found out using Differential Evolution (DE) algorithm. In this way, the side lobe levels have been reduced below -22 dB with fixed FNBW and below -44 dB without fixing FNBW. The obtained values of side lobe level using GA are -21.37 dB for fixed FNBW and -40.11 dB for variable FNBW.

In the second case as the computed value of d_m is greater than 0.5λ , so the total number of elements in the optimized arrays are also reduced.

Table 1. Maximum side lobe levels, FNBW and computation time for the optimized arrays with and without fixed FNBW computed individually using DE and GA.

| Types of array | DE | | | GA | | |
|---|-----------------------------|---------------|-----------------|------------------------------|---------------|-----------------|
| | Maximum sidelobe level (dB) | FNBW (degree) | Time (hr : min) | Maximum side lobe level (dB) | FNBW (degree) | Time (hr : min) |
| Optimized radial amplitude with fixed d_m (Fixed FNBW) | -22.077 | 14.800 | 2:08 | -21.22 | 14.800 | 2:17 |
| Optimized radial amplitude with optimized d_m (Fixed FNBW) | -22.079 | 14.800 | 1:56 | -21.37 | 14.800 | 2:09 |
| Optimized radial amplitude with fixed d_m (Variable FNBW) | -43.946 | 24.800 | 2:05 | -39.82 | 23.400 | 2:11 |
| Optimized radial amplitude with optimized d_m (Variable FNBW) | -44.070 | 25.000 | 1:54 | -40.11 | 23.200 | 2:13 |

All these simulations are performed using a PC having Intel core2 duo processor with 3 GHz clock frequency, 2 GB of RAM and Microsoft windows XP 32 bit operating system.

Table 1 shows the maximum side lobe level, FNBW and computation time for the optimized arrays with and without fixed FNBW computed individually using DE and GA. Table 1 also shows the relative improvements in the side lobe levels when optimum interelement distance is computed along with radial amplitude distributions keeping FNBW fixed or variable. From Table 1 it can be noticed that the array optimized using DE gives better side lobe levels than the optimized array using GA for all the cases.

