Error Rate Analysis of MIMO System with OSTBC and MRC in Composite Weibull-Gamma Fading

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Abstract: A paradigm for multiple-input multiple-output (MIMO) channel modeling demands diversity technique to compensate the effects of fading and shadowing. In this paper, error performance of MIMO system is analyzed with orthogonal space time block code (OSTBC) and maximal ratio combining (MRC) techniques over Weibull-gamma (WG) fading channel. WG distribution forms a composite channel model which is used to determine the effects of multipath as well as shadowing. In this scenario, exact analytical expressions for symbol error rate (SER) are computed for MIMO systems using OSTBC and MRC. Consequently, the effects of fading severity by varying shape parameters are also shown by simulation and analytical results.

Keywords: multiple-input multiple-output, orthogonal space time block code, maximal ratio combining, Weibull-gamma fading, symbol error rate.

1. Introduction

In the present wireless systems, the demand of proliferation of bandwidth for high data rate and reliable communication requirement is increasing. Multiple-input multiple-output (MIMO) systems have been investigated to achieve these requirements. Also, diversity techniques have been recommended for increasing the reliability of system. Orthogonal space time block codes (OSTBCs) have been employed in MIMO systems to achieve the full transmit diversity while permitting a simple maximum likelihood decoding algorithm. This algorithm is based on linear processing of received signals [1-2]. MIMO system uses receiver diversity e.g. maximal ratio combining to achieve improved error rate performance by maximizing output signal-to-noise ratio (SNR) [3].

Various channel models have been addressed in literature to model small scale fading and the system performance has been evaluated for Rayleigh, Rician, Nakagami-m, Weibull fading scenario with OSTBC using different modulation techniques [4-6]. In [7-9], error rate of MIMO-MRC system has been analyzed with and without noise and interference. In [10], computationally efficient bit error rate (BER) expressions have been computed for MIMO system using OSTBC and MRC with transmit antenna selection (TAS) technique in equicorrelated Rayleigh fading environment. It has been analyzed that TAS/MRC performs better than OSTBC in terms of BER. Large scale fading effect (i.e., shadowing) has also been considered along with small scale fading for analyzing the system performance. Hence, lognormal and gamma distributions have been introduced to evaluate shadowing effects [11-12]. In [13-15], channel models like Rayleigh-lognormal, Nakagami-m- lognormal and Weibulllognormal channels have been investigated. Although, gamma distribution is numerically more appropriate than lognormal distribution to model shadowing effects [16], several generic/composite distributions have been proposed using gamma distribution as an alternative of lognormal distribution. For example, K [16], and Generalized-K [17] channel models have been recommended for reducing complexity and enhancing accuracy of computational analysis.

In [12], a newly recommended composite fading channel i.e. Weibull-gamma (WG) has been described in the hypergeometric form and subsequently an alternative has been given in [18] where WG distribution is denoted in the form of Meijer-G function. In this composite fading, Weibull fading considers the radar clutters, different mobile fading effects and gamma fading

considers the effect of shadowing. In [19], outage probability has been obtained through a joint distribution using selection combining. In addition, arbitrary input branches have been considered over exponentially correlated composite WG fading channels. Thereafter, representations of bivariate WG distribution and outage probability with dual-branch selection diversity have been measured over bivariate WG fading channel [20].

The recent work proposed in [21-22] is the motivation behind this paper. However, the statistical characterization of MIMO-WG fading channel is not fully investigated in the existing literature. Therefore, this channel model is taken into consideration for the system performance analysis.

In this paper, WG channel is used with OSTBC and MRC for analyzing the MIMO system performance. Although, MRC gives the superior error rate performance compared to that of other receiver combining techniques, OSTBC is less complex than MRC. CDF based approach is used to evaluate the SER performance for the proposed system design. Exact analytical expressions for SER are computed in terms of Meijer-G function. Analytical results are verified by simulation results. As higher order modulation schemes offer high data rates, hence, 16-phase shift keying (16-PSK) and 16-quadrature amplitude modulation (16-QAM) are used to analyze the system performance. Consequently, the dominating effects of fading and shadowing with their small parameters values as well as the mitigating effects by varying their parameters values are demonstrated.

The remainder of this paper is prepared as follows. MIMO system and channel model are described in section 2. Analytical and simulation results for SER are given in section 3. Finally, the paper is concluded in section 4.

2. System and Channel Model

MIMO systems consider $N_r \times N_t$ antennas where, N_t and N_r indicate number of transmit and receive antennas respectively with complex channel matrix H. In H, each element of channel is independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and N_0 variance. The received signal over \mathbb{T} successive symbol durations is represented by Z, which is a complex matrix with size $N_r \times \mathbb{T}$

$$Z = \sqrt{\frac{E_s}{N_t}} H X + \mathcal{N} \tag{1}$$

where, E_s is total transmit energy at each time interval, X is transmit signal with matrix size $N_t \times \mathbb{T}$, \mathcal{N} is receiver noise with matrix size $N_r \times \mathbb{T}$ and its coefficients modeled as i.i.d. random variables.

The WG fading channel *H* is considered to model the effects of large and small scale fading. This channel matrix is given by $H = \sqrt{G/d^{\vartheta}}H_w$. Here, *d* is the distance between transmitter and receiver and ϑ is the path loss exponent. In [23], probability density function (PDF) of \mathcal{G} which denotes i.i.d. gamma random variable is given as

$$f(g) = \frac{1}{\Gamma m \,\Omega^m} \, g^{m-1} \exp\left(\frac{-g}{\Omega}\right), \qquad g, \Omega, m \ge 0 \tag{2}$$

Here, amplitude $r = |h_{i,j}|$ is Weibull distributed for each entry of H_w , $h_{i,j}$ is the channel gain among i^{th} transmit antenna and j^{th} receive antenna. Phase is uniformly distributed in the interval $[0, 2\pi]$. Each entry of matrix H_w follows an i.i.d Weibull PDF [23], which is given by

$$f(r) = \beta \left(\frac{\Gamma(1+2/\beta)}{g}\right)^{\beta/2} r^{\beta-1} \exp\left[-\left(r^2 \frac{\Gamma(1+2/\beta)}{g}\right)^{\beta/2}\right], \ \beta > 0, g \ge 0, r \ge 0$$
(3)

where, $g = E[r^2]$, $\Omega = E[g^2]$, m and β are gamma and Weibull fading parameters respectively. In [18], WG composite PDF is calculated from (2) and (3) through conditional PDF. For $m \rightarrow \infty$, the WG distribution approximates Weibull distribution. Using [23, eq. (9.34.3)], for $\beta = 2$, it follows K or Rayleigh-lognormal distribution. For $\beta = 2$, $m \rightarrow \infty$, it approaches Rayleigh distribution and for $\beta \to \infty$, $m \to \infty$, it approaches additive white Gaussian noise (AWGN) distribution. Weibull and gamma random variables are independent of each other.

Cumulative distribution function (CDF) of γ given in [18] is represented as

$$F_{\gamma}(\gamma) = \left(\frac{\Gamma\left(1+\frac{2}{\beta}\right)}{\frac{\bar{\gamma}}{m}}\right)^{\frac{\beta}{2}} \frac{\beta k^{\frac{1}{2}\ell}m^{-\frac{\beta+3}{2}}\gamma^{\frac{\beta}{2}}}{2\Gamma m(\sqrt{2\pi})^{\ell+k-2}} G_{1,k+\ell+1}^{k+\ell,1} \left(\frac{\left[m\Gamma\left(1+\frac{2}{\beta}\right)\gamma\right]^{\ell}}{(\bar{\gamma}\ell)^{\ell}k^{k}}\right|_{b_{k+\ell},-\frac{1}{k}}^{b_{k+\ell},-\frac{1}{k}}$$
(4)

where, $\frac{\ell}{k} = \frac{\beta}{2}$, $\Delta(u, v) = \frac{v}{u}$, $\frac{v+1}{u}$, ..., $\frac{v+u-1}{u}$, ℓ and k are positive integers. To enhance the system performance, OSTBC and MRC are employed which is given in section 2.1 and 2.2 respectively.

A. MIMO-OSTBC System

In space-time coded MIMO systems, mapping is done by converting number of bits into symbols [25]. The effective SNR (γ) for MIMO-OSTBC is obtained as

$$\gamma_o = \frac{\rho}{R_c N_t} \left[\text{trace}(\mathbf{H}^{\dagger} \mathbf{H}) \right]$$
(5)

where, $= E_s/N_0$, R_c denotes the code rate of OSTBC, $\|.\|$ represents the Frobenius norm of matrix, (.)[†] is Hermitian transposition and trace(.) is the summation of diagonal elements of matrix or matrix trace. trace $(H^{\dagger}H) = \|H\|^2 = \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} \|h_{ij}\|^2 = \mathcal{G} \operatorname{trace}(H_w^{\dagger}H_w)$.

B. MIMO-MRC System

According to [25], a weighted sum in MIMO-MRC system is defined by combining all the N_r branches and represented as

$$Y_{M} = (W_{1}^{M}, W_{2}^{M}, \dots, W_{N_{r}}^{M})Y = \sum_{j=1}^{N_{r}} W_{j}^{M}Y_{j}$$
(6)

where, Y denotes is the received signal, $W_{N_r}^M$ is weight vector and $(W_1^M, W_2^M, \dots, W_{N_r}^M) = W_M^T$.

The combined signal Y_j is represented by signal and noise vectors. Therefore, average SNR for MRC is expressed by the ratio of signal power p_{sig} to noise power p_{noise} , which is given as

$$\gamma_{M} = \frac{p_{sig}}{p_{noise}} = \frac{E_{s} |W_{M}^{T}H|^{2}}{N_{0} ||W_{M}^{T}||^{2}}$$
(7)

where, $H = [h_1 h_2 ... h_{N_r}]^T$, [.]^T is the transpose of matrix. By following Cauchy-Schwartz inequality from [25], average SNR illustrates the upper bound as

$$\gamma_M = \frac{E_S \|H\|^2}{N_0} \tag{8}$$

In section 3, (5) and (8) are used to calculate SER for efficient system design in WG fading environment.

3. Symbol Error Rate Analysis

SER is a substantial measure to evaluate the MIMO system performance. In [26], the average SER is given as

SER
$$(\rho) = E\rho[a_0 Q(\sqrt{2b_0 \rho})]$$
 (9)

where Q(.) is the Gaussian Q-function, a_0 and b_0 are explicitly defined modulation constants. The values of these constants for M-PSK and M-QAM are given in table 1. Keerti Tiwari, et al.

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	Modulation Technique	Modulation constants (a_0, b_0)
	M-PSK	$(2,\sin^2(\pi/M))$
	M-QAM	(4, 3/(2(M-1)))

Table 1. Modulation techniques with modulation-explicit constants

The appropriate representation of (9) can be expressed as

$$\operatorname{SER} = \frac{a_0 \sqrt{b_0}}{2\sqrt{\pi}} \int_0^\infty \frac{e^{-b_0 t}}{\sqrt{t}} F_\rho(t) dt \tag{10}$$

where, $F_{\rho}(t)$ is the CDF of ρ . Substituting (4) into (10), setting the value of γ_o and after some mathematical computation, the appropriate SER expression of (10) for proposed OSTBC-MIMO system is achieved using [24, eq. (7.813.1)]

$$\operatorname{SER}_{0}(\rho) = \frac{a_{0}b_{0}^{\frac{1}{2}-\frac{1}{\lambda}+\frac{1-\beta_{ij}}{2\lambda}}}{2\sqrt{\pi}} \left(\frac{R_{c} d^{\vartheta} N_{t} \Gamma\left(1+\frac{2}{\beta_{ij}}\right)}{\Omega_{ij} m_{ij} \rho}\right)^{\frac{\beta_{ij}}{2}} \frac{\frac{1}{k^{\frac{1}{2}} \lambda} m_{ij} - \frac{\beta_{ij}+3}{2}}{2\Gamma m_{ij} (\sqrt{2\pi})^{\lambda+k-2}}$$

$$\times G_{2,k+\lambda+1}^{k+\lambda,2} \left(\frac{\left[R_c \, d^\vartheta N_t \, m_{ij} \Gamma\left(1+^2/\beta_{ij}\right) \right]^\lambda}{\left(\Omega_{ij} m_{ij} \rho \lambda\right)^\lambda k^k b_0} \right|_{b_{k+\lambda}, -\frac{1}{k}}^{1-\frac{1}{\lambda} - \frac{\beta_{ij}-1}{2\lambda}, 1-\frac{1}{m_{ij}}} \right)$$
(11)

Similarly, substituting the value of γ_M , SER for MIMO-MRC system can be represented as

$$\operatorname{SER}_{M}(\rho) = \frac{a_{0}b_{0}^{\frac{1}{2}-\frac{1}{\lambda}+\frac{1-\beta_{j}}{2\lambda}}}{2\sqrt{\pi}} \left(\frac{\Gamma\left(1+\frac{2}{\beta_{j}}\right)}{\Omega_{j}m_{j}\rho} \right)^{\frac{1}{2}} \frac{k^{\frac{1}{2}}\lambda^{m_{j}}-\frac{\beta_{j}+3}{2}-1}}{2\Gamma m_{j}(\sqrt{2\pi})^{\lambda+k-2}} \times G_{2,k+\lambda+1}^{k+\lambda,2} \left(\frac{\left[m_{j}\Gamma\left(1+\frac{2}{\beta_{j}}\right)\right]^{\lambda}}{\left(\Omega_{j}m_{j}\rho\lambda\right)^{\lambda}k^{k}b_{0}} \right|_{b_{k+\lambda},-\frac{1}{k}}^{b_{j}-1} \right)$$
(12)

The analytical results can be obtained using (11) and (12). These expressions are applicable for an arbitrary number of antenna configurations and arbitrary fading scenarios.

Figures 1-4 demonstrate the system performance with the consideration of $N_r \times N_t$ as 1×2 and 1×4 for OSTBC and 2×1 and 4×1 for MRC over WG fading channel to signify the suitability of proposed computation. More explicitly, in Figure. 1 and 2, SER is plotted for MIMO-OSTBC for numerous values of β and m using 16-PSK and 16-QAM respectively. SER improves with the increase in ρ . Also, for higher values of β and m with a fixed ρ , SER performance increases. Consequently, improved SER is achieved by increasing N_t from 2 to 4. It is noted that for $\beta = 2$, WG channel model approximates K channel model. Although, higher order modulation provides high data rate, 16-PSK and 16-QAM modulation schemes are used to analyze the system performance. The number of constellation points is same in both the modulation techniques but 16-QAM gives improved error performance, it can be seen from Figures 1-4.



Figure 1. SER for MIMO-OSTBC (1×2 and 1×4) systems using 16-PSK over WG fading channel



Figure 2. SER for MIMO-OSTBC (1×2 and 1×4) systems using 16-QAM over WG fading channel



Figure 3. SER for MIMO-MRC (2×1 and 4×1) systems using 16-PSK over WG fading channel

It is clearly depicted from the results that 16-PSK gives 3 dB reduced error performance than 16-QAM. For 16-PSK, $a_0 = 2$, $b_0 = 0.0381$ and for 16-QAM, $a_0 = 4$, $b_0 = 0.1$. For simulation, $\Omega = 1$ and $R_c = 1$ is considered.

Multipath is highly accountable to degrade the wireless system performance. The impact of shadowing can also reduce the error rate performance of wireless system. Hence, reduced shadowing effect can be observed by high values of m. Here, four combinations of β and m are considered and SER performance is evaluated at these values.



Figure 4. SER for MIMO-MRC (2×1 and 4×1) systems using 16-QAM over WG fading channel

The WG channel matrix *H* is created by the product of the each entries of Weibull and gamma random variable. In Figure 3 and 4, SER performance of MIMO-MRC system is evaluated with the same configurations and parameters as considered for OSTBC. It is analyzed from Figures. 1-4 that MRC (2×1 and 4×1) gives approximately 3 dB enhanced SER performance than OSTBC (1×2 and 1×4). This 3dB loss is obtained due to the radiation of half energy by each transmit antenna to provide the total radiated power equivalent to one transmit antenna. By changing the antenna configuration or increasing the number of antennas at transmitter and receiver end, SER performance can be highly improved but with the increased cost of system.

5. Conclusion

In this paper, WG channel model is presented to analyze the MIMO system performance. MIMO system is investigated with OSTBC and MRC in terms of SER operating over WG fading channels. Simulation and analytical results illustrate that 16-QAM gives decreased error rate than 16-PSK under this scenario. Also, MRC shows the better error performance than OSTBC. The mathematical and simulation analysis provide the worth and flexibility of proposed channel model. Further, this work can be extended for OFDM with multiple antenna system to measure the error rate and capacity performance in diversity scenario over WG fading channel.

6. References

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Error Rate Analysis of MIMO System with OSTBC and MRC in Composite



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