# Coherent Modulation Analysis of Photoplethysmographic Signals by Time-varying Filterbank

Richard Mengko<sup>1</sup>, Habibur Muhaimin<sup>2</sup>, and Tati Latifah R. Mengko<sup>3</sup>

Biomedical Engineering Research Group Institut Teknologi Bandung – ITB, Bandung, Indonesia <sup>1</sup>richard@mengko.com, <sup>2</sup>habibur@stei.itb.ac.id, <sup>3</sup>tati@stei.itb.ac.id

*Abstract*: In spite of its usefulness to early detect cardiovascular disease, photoplethysmographic (PPG) signals are prone to motion artifacts due to its typical measurement locations. These artifacts consequently affect the reliability of systolic peak detection-based PPG parameter calculations. In this paper, the PPG signals are modeled by a small number (3 to 6) of Amplitude-Frequency Modulation (AM-FM) modulated signals such that the synthesis signal preserves its harmonics structure in time-frequency domain. We propose a coherent AM modulating signal detection by which the filterbank tracks Instantaneous Frequency (IF) of each harmonic component. This time-varying filterbank method offers bandwidth preservation of the AM components. Furthermore, the experimental results show that motion artifact (MA) noise reference for the input of the adaptive filter is generated from the corrupted PPG signal itself by using one of applications of our method.

*Index Terms:* AM-FM decomposition, modulation filtering, photoplethysmographic signals, time-frequency analysis, time-varying filterbank.

# 1. Introduction

A photoplethysmogram (PPG) system measures volume changes of blood in peripheral cardiovascular system, such as fingertips and ear lobes. PPG-based monitoring systems are widely used in early disease detection due to its non-invasive technique characteristic and easy to measure advantage. However, the quality of the PPG signal heavily depends on the motion of patients. In practice, without any mitigations, patients need to be assured to keep his finger position steady such that the PPG sensor is attached firmly.

The major frequency band of uncorrupted PPG power spectra was reported depending on the subject's heart rate. In [1] at most 95% of the signal power lies on the frequency band up to three times the subject's heart rate. The more general information about the frequency band of the PPG signal varies between 0.6-15 Hz [2], 0.05-8 Hz [3], and 0.5-4 Hz [4]. However, the frequency band of typical motion artifacts overlaps with of the PPG signal. For examples, frequency ranges higher than 0.1 Hz, between 1-2.5 Hz, and between 0.04-1.6 Hz are frequency band due to tiny motion [4], daily motion, such as walking and jogging [5], and respiration [4], respectively.

The simplest mitigation due to motion artifacts involves a combination of low pass filters. A conventional low pass filter to a time-domain signal is avoided since the frequency band of motion artifacts overlaps with the frequency band of the PPG signal. Periodic moving average filters [4, 6] performed better than a conventional sample-by-sample moving average filters. However, this method generalizes the characteristics of the non-stationary PPG signal.

One of other motion artifacts mitigations includes more than one measured signal. A recursive least square method-based adaptive filter with a reference signal from an accelerometer was developed by Asada et al. [7]. It has been shown that there is correlation between acceleration of the motion and the motion-corrupted PPG signal. Reduction of motion artifacts based on Independent Component Analysis (ICA) assumes that motion artifacts and the PPG signal have little correlation [8]. This method requires at least two measured PPG signals in order to estimate the uncorrupted PPG signal. The inverse mixing matrix is estimated by fast ICA algorithm [9].

Speech signals are often modeled by a collection of time-varying low frequency modulators that modulates time-varying higher frequency carriers [10, 11]. An analysis of psychoacoustics was related to the effects of temporal smearing on the modulators to find the most sensitive modulation frequencies to speech intelligibility [12]. Other parameters of speech signals, such as harmonics and pitch, are embedded in their sub-band signal components [13].

Harmonics structures of the PPG signal enable signal analysis similar to voiced speech signals in a time-frequency domain. In this paper, we analyze the PPG signal to highlight its harmonics structures and reduce the motion artifacts by filtering of the insignificant modulation frequency. Firstly, we decompose the PPG signal to a set of narrowband Amplitude-Frequency Modulation (AM-FM) modulated carriers by a filterbank whose centre frequencies track Instantaneous Frequencies (IFs) of the harmonic components. Secondly, the AM modulating signals are coherently detected after the FM modulated carriers have been estimated from the narrowband signals. Thirdly, the complex-valued AM modulating signals are filtered depending on the applications. Lastly, the modulation filtered narrowband signals are formed by combining the filtered AM signals with the FM modulated carriers and, then, the PPG signal is synthesized.

The following sections are arranged as follows. Section 2 explains the theory of AM-FM signal decompositions. The incoherent AM detector is compared with the proposed coherent method dedicated for specific harmonics signals, such as the PPG signals. The harmonics tracking as an important element of the proposed method is described with focus on controlling the frequency shift of the reference low pass filter. Section 3 shows the results of signal analysis and the corresponding discussion mainly based on modulation filtering projection test. Some comments on the performance is given with comparison to the IF estimation by time-varying spectral Centre of Gravity (CoG). Finally, conclusions are given in Section 4.

#### 2. Theory

#### A. Signal Models

Let x[n] be a discrete-time PPG signal that is considered as a non-stationary broadband signal. Outputs of a baseband uniform filterbank  $X^{b}[k,n]$  is defined as lowpass-filtered signals of a down-shifted broadband signal that are stacked to form a two-dimensional signal as on (1),

$$X^{b}[k,n] = \sum_{m=n-M/2}^{n+M/2} x_{k}[m]w^{b}[m-n]$$

$$= x_{k}[n] * w^{b}[-n]$$
(1)

where  $x_k[n]$ , as shown on (2), is defined as the down-shifted broadband signal at channel k with N is the number of filters and  $w^b[n]$  is an impulse response of a finite length lowpass filter whose its non-zero values are at n = -M/2, ..., M/2 [14]. The  $X^b[k,n]$  consists of oversampled baseband signals and, thus, might be able to be down-sampled by a factor of integer R as on (3). Otherwise stated, the discussion of the implementation is limited to the oversampled case. To guarantee a perfect reconstruction of the broadband signal, the impulse response  $w^b[n]$  needs to satisfy (4) [14].

$$x_k[n] = x[n]e^{-j\frac{2\pi k}{N}n}, \ k = 0, 1, 2, ..., N-1$$
(2)

$$X^{b}[k,nR] = X^{b}[k,l]$$
(3)

$$w^{b}[rN] = 0, \text{ for } |r| = 1, 2, ...$$
 (4)

An alternative calculation of (1) is provided by an Overlap-Add (OLA) analysis method such that these two methods are a duality of short-time Fourier Transform (STFT) calculation [14]. Given the calculation of STFT on (5), the OLA analysis method rearranges the equation

into (6). In the OLA point of view, the requirement on (4) loosely states that the length of windowed incoming signal should be less than the N-point DFT.

$$X^{b}[k,n] = \sum_{m=n-M/2}^{n+M/2} x[m]w^{b}[m-n]e^{-j2\pi\frac{k}{N}m}$$
(5)  
$$X^{b}[k,n] = \sum_{m=n-M/2}^{n+M/2} x[m]w^{b}[m-n]e^{-j2\pi\frac{k}{N}m}; p = m-n$$
$$= e^{-j2\pi\frac{k}{N}n} \sum_{p=-M/2}^{M/2} x[p+n]w^{b}[p]e^{-j2\pi\frac{k}{N}p}$$
(6)  
$$= e^{-j2\pi\frac{k}{N}n} \text{DFT} \{x[p+n]w^{b}[p]\}$$

Each trajectory signal on channel k in the baseband uniform filterbank  $X^{b}[k,n]$ , that is  $X_{k}^{b}[n]$ , can be modulated to passband by (7). This calculation of a passband uniform filterbank, as shown on (8), represents the broadband signal that is bandpass filtered by modulated lowpass filters  $w_{k}[n]$ . Another way to calculate (8) is shown on (9) in which  $w_{k}[n]$  is defined on (10).

$$X[k,n] = X^{b}[k,n]e^{j2\pi\frac{k}{N}n}$$
(7)  

$$X[k,n] = \sum_{m=n-M/2}^{n+M/2} x[m]w^{b}[m-n]e^{-j2\pi\frac{k}{N}(m-n)}$$
(8)  

$$= \sum_{m=n-M/2}^{n+M/2} x[m]w_{k}[n-m]$$
(9)  

$$= x[n] * w_{k}[n]$$

$$X[k,n] = \sum_{m=n-M/2}^{n+M/2} x[m]w^{b}[m-n]e^{-j2\pi\frac{k}{N}(m-n)}; p = -(m-n)$$

$$= \sum_{p=-M/2}^{M/2} x[n-p]w^{b}[-p]e^{j2\pi\frac{k}{N}p} = \sum_{p=-M/2}^{M/2} x[n-p]w_{k}[p]$$
(9)  

$$= x[n] * w_{k}[n]$$
(9)

Since a causal filter  $w_k[n]$  is needed for more practical implementation, the impulse response of a finite lowpass filter  $w^b[n]$  is made to be an anti-causal signal whose its non-zero values span from n = -M to n = 0. Thus, the passband uniform filterbank X[k,n] on (9) is altered to be (11). The mechanical description of the causal implementation is provided by Figure 1.

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Figure 1. Description of calculation of narrowband signals

$$X[k,n] = \sum_{m=n-M}^{n} x[m] w^{b}[m-n] e^{-j2\pi \frac{k}{N}(m-n)}; p = -(m-n)$$
  
$$= \sum_{p=0}^{M} x[n-p] w^{b}[-p] e^{j2\pi \frac{k}{N}p} = \sum_{p=0}^{M} x[n-p] w_{k}[p]$$
  
$$= x[n] * w_{k}[n]$$
(11)

The broadband signal x[n] is synthesized by a summation operator as given by (12). By including the amplitude scaling for normalization 1/N, the synthesis equation on (12) is an inverse STFT equation.

$$x[n] = \sum_{k=0}^{N-1} X[k,n]$$
  
=  $\sum_{k=0}^{N-1} X^{b}[k,n] e^{j2\pi \frac{k}{N}n}$  (12)

Since the baseband uniform filterbank  $X^{b}[k,n]$  is a two-dimensional complex-valued signal, the synthesis equation might be represented by the magnitude and phase as on (13).

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} \left| X^{b}[k,n] \right| \cos(2\pi \frac{k}{N} n + \angle X^{b}[k,n]) + j \left| X^{b}[k,n] \right| \sin(2\pi \frac{k}{N} n + \angle X^{b}[k,n])$$
(13)

Moreover, if x[n] is a real-valued time domain signal, the summation of the imaginary parts on (13) is always zero and, therefore, is simplified to (14). Furthermore, x[n] can be represented only by its positive frequency components with scalings due to both even symmetry of  $|X^{b}[k,n]|$  and odd symmetry of  $\angle X^{b}[k,n]$  as on (15).

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} |X^{b}[k,n]| \cos(2\pi \frac{k}{N} n + \angle X^{b}[k,n])$$
(14)

$$x[n] = \frac{2}{N} \sum_{k=1}^{N/2-1} \left| X^{b}[k,n] \right| \cos(2\pi \frac{k}{N} n + \angle X^{b}[k,n]) + \frac{1}{N} \sum_{k=0,N/2} \left| X^{b}[k,n] \right| \cos(2\pi \frac{k}{N} n + \angle X^{b}[k,n])$$
(15)

Different scalings on (15) can be avoided by considering an analytic time domain signal  $x_{+}[n]$  as an input to (11) to calculate the passband uniform filterbank counterpart  $X_{+}[k,n]$ . The relationship between  $X_{+}[k,n]$  and X[k,n] is given by (16). The analytic time domain signal  $x_{+}[n]$  is related to x[n] through (17), where  $H\{x[n]\}$  is to filter x[n] by a Hilbert filter. Thus, the synthesis equation is alternatively given by (18).

$$X_{+}[k,n] = \begin{cases} 2X[k,n], \ k = 1,...,N/2 - 1\\ X[k,n], \ k = 0,N/2\\ 0, \ k = N/2 + 1, N - 1 \end{cases}$$
(16)

$$x_{+}[n] = x[n] + jH\{x[n]\}$$
(17)

$$x[n] = \frac{1}{N} \sum_{k=0}^{N/2} X_{+}[k,n]$$

$$= \frac{1}{N} \sum_{k=0}^{N/2} \left| X_{+}^{b}[k,n] \right| \cos(2\pi \frac{k}{N} n + \angle X_{+}^{b}[k,n])$$
(18)

A signal model on (18) mathematically describes that each trajectory k of narrowband signals  $X_{+}[k,n]$ , that is  $X_{+k}[n]$ , might be modeled as both AM-FM modulated signals. In general, real-valued time domain signal x[n] can be modeled by a summation of AM-FM modulated signals  $y_k[n]$  as given by (19), where L is the number of channels,  $m_k[n]$  is AM modulating signals, and  $c_k[n]$  is FM modulated carriers.

$$x[n] = \Re\left\{\sum_{k=0}^{L} y_k[n]\right\}$$

$$= \Re\left\{\sum_{k=0}^{L} m_k[n]c_k[n]\right\}$$
(19)

There are unlimited methods to decompose signals to their AM and FM components. In other words, given  $y_k[n]$ , finding  $m_k[n]$  and  $c_k[n]$  (or signal that modulates the frequency of the carrier signal) is not a one-to-one function. In order to be a valid AM-FM decomposition, the AM modulating signal and the FM modulating signal need to satisfy some requirements. Since the narrowband signal  $y_k[n]$  to be modeled is band-limited, both the modulating signals also need to be band-limited. Such decompositions are called bandwidth preserved decompositions [10]. We compare two methods to find the AM-FM components in the following sections. Incoherent detector calculates the AM components without having to firstly estimate their FM components. On the other hand, coherent detector finds the AM components if their FM components have been estimated first. We propose a coherent detector whose bandpass filter to calculate the narrowband signal tracks the Instantaneous Frequency (IF) of the output of the filter.

#### A.1 Incoherent Detector

Given  $y_k[n]$ , that is the output of the passband filterbank, incoherent detector calculates the modulator and carrier signals by its magnitude and phase respectively as shown on (20) and (21).

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$$m_k[n] = \left| y_k[n] \right| \tag{20}$$

$$c_k[n] = e^{j \angle y_k[n]} \tag{21}$$

Thus, this searching for the AM and FM components might be independent to each other. For example, the derivation of the signal model on (18) for L = N/2 shows that

$$m_{k}[n] = \left| X_{+}[k,n] \right| / N = \left| X_{+}^{b}[k,n] \right| / N \text{ and } c_{k}[n] = e^{j \angle X_{+}[k,n]} = e^{2\pi \frac{\lambda}{N}n + \angle X_{+}^{b}[k,n]}.$$
(22)

Greater flexibility in choosing some parameters of the model is offered by phase vocoder [11] by which the number of filters to compose the filterbank, the bandwidth of each filter, and the usage of possible smoothing filters are left to the applications and types of signals. In continuous time, the signal model of the phase vocoder is given by (23), where h(t) is the impulse response of a lowpass filter,  $X(\omega_k, t)$  is the STFT of x(t), and  $\theta_k(t)$  is the phase of the narrowband lowpass signals. Generally  $\theta_k(t)$  is not bounded in frequency, yet its time-derivative  $\varphi_k(t) = \dot{\theta}_k(t)/2\pi$ , named Instantaneous Frequency (IF), is more well-behaved [11]. Therefore, the signal model on (23) becomes (24) in which  $\hat{\theta}_k(t)$  is written instead of  $\theta_k(t)$  due to the acceptable loss of additive constant from the derivation.

$$\begin{aligned} x(t) &= \sum_{k=1}^{L} x(t) * g_{k}(t) \\ &= \sum_{k=1}^{L} x(t) * \left(h(t) \cos \omega_{k} t\right) = \sum_{k=1}^{L} \left| X(\omega_{k}, t) \right| \cos \left(\omega_{k} t + \angle X(\omega_{k}, t)\right) \end{aligned}$$
(23)  
$$&= \sum_{k=1}^{L} m_{k}(t) \cos \left(\omega_{k} t + \theta_{k}(t)\right) \\ x(t) &= \sum_{k=1}^{L} m_{k}(t) \cos \left(\omega_{k} t + \hat{\theta}_{k}(t)\right) \\ &= \sum_{k=1}^{L} m_{k}(t) \cos \left(\omega_{k} t + \int_{0}^{t} \dot{\theta}_{k}(\tau) d\tau\right) = \sum_{k=1}^{L} m_{k}(t) \cos \left(\omega_{k} t + 2\pi \int_{0}^{t} \varphi_{k}(\tau) d\tau\right) \end{aligned}$$
(24)

Shown on (25), the AM and FM modulating signals are calculated from the outputs of lowpass filterbank similar to (1), whose the real and imaginary parts are denoted by  $a(\omega_k,t)$  and  $-b(\omega_k,t)$  respectively. The discrete implementation of (25) is shown on (26) in which  $\Delta a[k,n] = a[k,n] - a[k,n-1]$  and  $\Delta b[k,n] = b[k,n] - b[k,n-1]$ . Therefore, the broadband signal can be reconstructed by (27).

$$m_k(t) = \sqrt{a^2(\omega_k, t) + b^2(\omega_k, t)}$$

$$\varphi(\omega_{k},t) = \frac{1}{2\pi} \dot{\theta}(\omega_{k},t) = \frac{1}{2\pi} \frac{d\theta(\omega_{k},t)}{dt}$$

$$= \frac{1}{2\pi} \frac{b(\omega_{k},t) da(\omega_{k},t)/dt - a(\omega_{k},t) db(\omega_{k},t)/dt}{a^{2}(\omega_{k},t) + b^{2}(\omega_{k},t)}$$
(25)

$$m_{k}[n] = \sqrt{a^{2}[k,n] + b^{2}[k,n]}$$

$$\varphi_{k}[n] = \frac{f_{s}}{2\pi} \frac{b[k,n]\Delta a[k,n] - a[k,n]\Delta b[k,n]}{a^{2}[k,n] + b^{2}[k,n]}$$
(26)

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$$x[n] = \sum_{k=1}^{L} m_k[n] \cos\left(\omega_k n + \frac{2\pi}{f_s} \sum_{l=0}^{n} \varphi_k[l]\right)$$
(27)

The signal components  $m_k[n]$  and  $\varphi_k[n]$  might be forced to be band-limited by filtering before the reconstruction to satisfy the bandwidth preservation requirement. However, as studied by Schimmel et al. on [10], filtering of  $m_k[n]$ , known as modulation filtering, rarely guarantees that the output is still at the same AM modulator domain since  $m_k[n]$  is defined in domain of positive real numbers. Some mitigations were applied by a rectifier and power compression [15]. An AM detector that keeps the domain of AM modulating signal after the filtering is called closed under convolution [10].

FAME method [16] is different from the phase vocoder in the analysis signal to find the AM and FM components. In phase vocoder, the problem statement can be summarized by given complex-valued baseband equivalent narrowband signal,

$$y_k^b(t) = m_k(t) \cos\left(2\pi \int_0^t \varphi_k(\tau) d\tau\right) - jm_k(t) \sin\left(2\pi \int_0^t \varphi_k(\tau) d\tau\right)$$
$$= a(\omega_k, t) - jb(\omega_k, t)$$

calculate  $m_k(t)$  and  $\varphi_k(t)$ , which discrete solutions have been given on (26). In the FAME method, the problem statement is from passband narrowband signal  $y_k(t)$  on (28), calculate  $m_k(t)$  and  $\varphi_k(t)$ . The slowly varying envelope  $m_k(t)$  is found from  $y_k(t)$  by a sequence of full-wave rectifier and LPF. On the other hand, the slowly varying frequency modulation is calculated from a sequence of IQ demodulation and IF estimation as on (25) followed by LPF to limit the frequency rate of the IF. Both calculations are depicted on Figure 2.

$$y_k(t) = m_k(t)\cos\left(\omega_k t + 2\pi \int_0^t \varphi_k(\tau) d\tau\right)$$
(28)

Summarized by (29), the incoherent detector assumes that the AM components have positive real values, while the carrier signal always has unit magnitude, consequently  $m_k(t) = |y_k(t)|$  and  $c_k(t) = e^{j \angle y_k(t)}$ , where  $\angle y_k(t) = \omega_k t + 2\pi \int_0^t \varphi_k(\tau) d\tau$ .

$$y_{k}(t) = m_{k}(t)c_{k}(t)$$

$$|y_{k}(t)|e^{j \angle y_{k}(t)} = |m_{k}(t)|e^{j \angle m_{k}(t)}|c_{k}(t)|e^{j \angle c_{k}(t)}$$
(29)



Figure 2. Block diagram of FAME method to calculate AM and FM modulating signals [16]

#### A.2 Coherent Detector

Coherent detector endeavors to find the AM modulating signal after having estimated the FM modulating carrier. In other words, the estimated carrier  $c_k(t)$  is used to extract the AM modulating signal  $m_k(t)$  from the narrowband signal  $y_k(t)$ . While the magnitude of the

estimated carrier is kept unitary, the coherent detector allows complex values in  $m_k(t)$ . The phase of the complex baseband envelope  $\angle m_k(t)$  is the remaining phase in the  $y_k(t)$  after substraction from the coherent carrier estimation as shown on (30).

$$m_{k}(t) = y_{k}(t)/c_{k}(t) = y_{k}(t)c_{k}^{*}(t)$$

$$m_{k}(t) = |y_{k}(t)|e^{j(\angle y_{k}(t)-\angle c_{k}(t))}$$
(30)

Harmonics signals, such as PPG signals, have clear *changes* of harmonic lines in timefrequency display. Tracking the harmonic frequencies to estimate the centre frequencies of the carriers is proposed to reduce the abrupt changes of the IF estimate. Moreover, by having limited the applications of the decomposition to harmonics signals, the proposed method uses less number of filters to form the filterbank. Each filter might have narrower bandwidth so that the approximation of the IF for the carrier estimation of each harmonic component is improved. By adapting the centre frequencies of the filters depending on the IF estimate, the proposed method focuses the signal modeling on limited frequency band where the centres of significant energy exist.

The narrowband signal demodulation enables two possible solutions depending on whether the narrowband signal is analytics or not. The real narrowband signal demodulation is similar to the FAME method except for that the frequency of the reference carrier signal now changes depending on the output of the tracking algorithm. The IQ demodulation by the tracking filter for real narrowband signal is given by (31), followed by lowpass filtering to yield the inphase and quadrature of AM modulating signals  $m_k^I(t)$  and  $m_k^Q(t)$  as on (32). The phase of the analytics bandpass signal  $\omega_k(t) + \theta_k(t)$  on (33) is now more band-limited due to the bandpass filtering that follows up its previous IF estimation.

$$y_{k}(t) = m_{k}(t)\cos\left(2\pi\int_{0}^{t}\left(\frac{\dot{\omega}_{k}(\tau)}{2\pi} + \varphi_{k}(\tau)\right)d\tau\right) = m_{k}(t)\cos\left(\omega_{k}(t) + \theta_{k}(t)\right)$$

$$y_{k}(t)\cos(\hat{\omega}_{k}(t)) \approx \frac{m_{k}(t)}{2}\cos\left(\hat{\theta}_{k}(t)\right) + \frac{m_{k}(t)}{2}\cos\left(\omega_{k}(t) + \theta_{k}(t) + \hat{\omega}_{k}(t)\right)$$

$$y_{k}(t)\sin(\hat{\omega}_{k}(t)) \approx -\frac{m_{k}(t)}{2}\sin\left(\hat{\theta}_{k}(t)\right) + \frac{m_{k}(t)}{2}\sin\left(\omega_{k}(t) + \theta_{k}(t) + \hat{\omega}_{k}(t)\right)$$

$$m_{k}^{t}(t) = \frac{m_{k}(t)}{2}\cos\left(\hat{\theta}_{k}(t)\right)$$

$$m_{k}(t) = |y_{k}(t)|e^{j(\omega_{k}(t) + \theta_{k}(t) - 2c_{k}(t))}$$

$$= |y_{k}(t)|e^{j(\omega_{k}(t) + \theta_{k}(\tau))d\tau - 2\pi\int_{0}^{t}\frac{\hat{\omega}_{k}(\tau)}{2\pi}d\tau}$$
(33)
$$= |y_{k}(t)|e^{j\hat{\theta}_{k}(t)}$$

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#### B. Coherent Detector for Harmonics Signal

The proposed system of coherent detector specifies the number of bandpass filters L according to the number of significant harmonic components. As described on Figure 3, the system consists of a collection of time-varying bandpass filters followed by the demodulation of the narrowband signals  $y_k[n]$  by FM modulated carriers  $c_k[n]$ . At each channel, the AM modulating signal  $m_k[n]$  is filtered to suppress the unwanted envelope frequency  $f_m$ . This process is termed as modulating filtering [10]. The FM modulated carrier is kept synchronised with the filtered AM modulating signal  $\tilde{m}_k[n]$  by delay compensation to form the modulation filtered narrowband signal  $\tilde{y}_k[n]$ .





The broadband signal x[n] is made analytics prior to the calculation of the narrowband signal  $y_k[n]$ . The convolutional block to calculate  $y_k[n]$  is similar to (11) except that now the coefficients of the FIR filters change every time index *n*,

$$y_{k}[n] = \sum_{p=0}^{M} x_{+}[n-p]w_{kn}[p]$$
(34)

where  $w_{kn}[p]$  is the value of the  $p^{\text{th}}$  coefficient of  $k^{\text{th}}$  bandpass filter at time index *n*. The FM modulated carrier  $c_k[n]$  is found through (35) where  $\omega_k[n]$  results from the harmonics tracking, while subsequently the AM modulating signal  $m_k[n]$  is coherently calculated from (30). Thus, each narrowband signal  $y_k[n]$  is modeled by single carrier whose frequency changes depending on the frequency of the harmonics.

$$c_k[n] = e^{j\omega_k[n]} \tag{35}$$

The synthesis of the broadband signal follows (19) by which

$$x[n] = \sum_{k=0}^{L} m_{k}^{\prime}[n]c_{k}^{\prime}[n] - m_{k}^{Q}[n]c_{k}^{Q}[n]$$

$$= \sum_{k=0}^{L} m_{k}^{\prime}[n]\cos\omega_{k}[n] - m_{k}^{Q}[n]\sin\omega_{k}[n].$$
(36)

The harmonics tracking that is used to calculate both the filter coefficients and the carrier signal, as well as the filtering block of the AM modulating signal, is given in the following sections.

#### **B.1** Harmonics Tracking

The analytics passband signal  $y_k[n]$  is found by filtering the broadband signal x[n] with a response impulse of a bandpass filter whose centre frequency  $\omega_k[n]$  changes depending on previous centre frequency and IF of the  $y_k[n]$ . Initially placed uniformly, the centre frequency in continuous time follows the system provided on (37) [13]. The estimated implementation in discrete time is given on (38) by which one might consider that the system is indeed an IIR filter as described by the difference equation with  $\varphi_k[n]$  and  $\omega_k[n]$  are the input and output of the filter respectively. Thus, a small constant a is chosen such that the transfer function of the filter, as given on (39), is stable.

$$\frac{d}{dt}\omega_k(t) = a\big(\varphi_k(t) - \omega_k(t)\big) \tag{37}$$

$$\omega_{k}[n+1] = a/f_{s} 2\pi \varphi_{k}[n] + (1 - a/f_{s})\omega_{k}[n]$$
(38)

$$H(z) = \frac{a/f_s 2\pi z^{-1}}{1 - (1 - a/f_s) z^{-1}}$$
(39)

The calculated centre frequency, then, is used to update the shifting property of the modulated lowpass filter on (10). The bandpass filter is rewritten on (40) to highlight the updating process at time n+1, where p = 0, 1, ..., M. The Instantaneous Phase (IP) at time n is estimated by (41) from which the IF  $\varphi_k[n]$  is calculated as on (42).

$$w_{k(n+1)}[p] = w^{b}[-p]e^{j\omega_{k}[n+1]p}$$
(40)

$$\theta_k[n] = unwrap \left\{ \angle y_k[n] \right\}$$
(41)

$$\varphi_k[n] = \frac{f_s}{2\pi} \left( \theta_k[n] - \theta_k[n-1] \right) \tag{42}$$

#### **B.2** AM Modulation Filtering

After decomposing the passband signal  $y_k[n]$  into the estimated carrier  $c_k[n]$  and the AM modulating signal  $m_k[n]$ , the performance test filters the AM modulating signal  $m_k[n]$  by linear phase (odd-length *P*) LPF  $h_m[n]$  and resynthesizes the AM filtered passband signal  $\tilde{y}_k[n]$  by a multiplication as shown on (43). The carrier signal  $c_k[n]$  is left unchanged except for a delay compensation introduced by the linear-phase filter on  $m_k[n]$ .

$$\tilde{y}_{k}[n] = (m_{k}[n] * h_{m}[n])(c_{k}[n] * \delta[n - (P - 1)/2])$$
  
=  $\tilde{m}_{k}[n]\tilde{c}_{k}[n]$  (43)

### 3. Results and Discussion

### A. Experiment Setup

A transmission mode PPG sensor was placed in either index or middle finger of subjects. The 940-nm infrared light source illuminates the finger at one side, while at the other side a photodetector records the incoming transmitted light intensity. Simple analog filters were constructed to pass frequency components between 0.1 and 3.4 Hz whose gain at passband was set 33.6 dB. Additionally, an analog manual potentiometer was attached to the output of the filters to adjust the amplitude of the filtered signal over different subject's characteristics. Furthermore, a 10-bit analog-to-digital converter sampled the analog signal with 1000 Hz sampling rate that further digitally down sampled to 100 Hz.

Utilizing the above measurement setup, 13-PPG recordings were obtained from one subject under 7-different measurement conditions, i.e. normal (relaxed, unmoved) finger, intermittently-pressed finger, intermittently-moved finger in vertical direction, intermittentlylifted arm, and intermittently-cuffed arm (each of which was recorded for 3 minutes); combinations of deep-fast-hold breathings (for 5 minutes), and induced reactive hyperemia (5 minutes of tightly-cuffed arm followed by 5 minutes of released cuff). The purpose of varying the measurement conditions is to simulate different degrees of artifacts expected to be handled by our developed algorithm.

The performance of the coherent modulation analysis introduced in this paper is demonstrated by the use of the least-mean square (LMS)-based adaptive filter for motion artifact (MA) reduction in PPG signals. We based our MA reduction from [17], where either FFT, SVD, or ICA was chosen to generate the synthetic noise reference for the LMS-based adaptive filter algorithm. However, in our work, we utilize the coherent modulation analysis for the noise reference instead of the three mentioned alternatives. The use of coherent modulation analysis was motivated by the difference in the PPG signal recording specification from the original LMS paper, which made the FFT-based analysis setup introduced in [17] inapplicable to our data. Another reason was the quasi-periodic nature of the actual PPG signals, which renders the SVD method unsuccessful in determining the correct signal periods. The SVD method relies on the determination of the singular value ratio (SVR) derived from the two largest singular values. For periodic signals, SVRs are generally very high, leading to correct determination of signal period; however for quasi-periodic signals corrupted by baseline drift due to the motion artifacts, SVRs become significantly lower, resulting in inaccurate estimation of signal period. Furthermore, in typical cases using our data, the SVD method, when correctly estimates the periodicity, evokes discontinuity between frames that construct the SVD matrix whose singular values have been altered. The detailed explanation on the incorporation of coherent modulation analysis in the LMS-based motion artifact reduction is given in part C of this section.

## B. Coherent Modulation Performances

The result of harmonics tracking is given by Figure 4 on which the centre frequencies of the filters are overlaid with time-frequency magnitude display. The adjacent centre frequencies tend to draw near the local maximum power as shown at the early time of the display on Figure 4 (a). To avoid such a converging occurrence, Abe et al. [13] suggested controlling capability based on a prescribed collection of thresholds of several parameters. The controlling parameters are the distance between the adjacent centre frequencies, the amplitude of the output of the filters, and the duration within which the output of the filters is above the threshold. Instead, by considering the harmonics structure of the PPG signals, we choose to control the allowable range of frequency displacement by some minimum and maximum frequency distance between the harmonic components. If the difference between the adjacent centre frequencies are equal to the previous time index. These minimum and maximum thresholds, along with the initial placement of centre frequencies of the filters, were calculated in advance

based on the calculation of average fundamental frequency by the average magnitude difference function,

$$E[\tau] = \frac{1}{N} \sum_{n=0}^{N-1} |x[n] - x[n+\tau]|.$$
(44)

frequency (Hz)



Figure 4. Harmonics tracking by Instantaneous Frequency (IF) estimation. (a) The number of filters (*L*) and initial spacing between filters are set independent to the characteristics of the signal, i.e. initial spacing between the filters are uniformly separated by
 Δω<sub>k</sub>[0]=ω<sub>k+1</sub>[0]-ω<sub>k</sub>[0]=Dynamic Frequency Range/L-1, where Dynamic Frequency Range is maximum analysis frequency, compared to (b) in which the number of filters and the initial spacing between filters depend on the average fundamental frequency.

Figure 4 (b) shows the result of the harmonics tracking with the number of filter (*L*) is set to 6 such that the bandpass filter  $w_6[p]$  tracks the 6<sup>th</sup> harmonic component and the minimum and maximum frequency distance are 0.88 and 1.32 multiple of the first harmonic. By this strategy, the dynamic frequency range is from around the 1<sup>st</sup> to the *L*<sup>th</sup> harmonic component and the harmonics structure of the PPG signal is preserved.

To assess the performance of such AM-FM decompositions, we follow a scenario by a modulation filtering projection test as on [18, 19]. The performance test compares the filtered AM modulating signal  $\tilde{m}_k[n]$  and recovered AM modulating signal  $\hat{m}_k[n]$  with reference to the original AM modulating signal  $m_k[n]$ . Figure 5 shows the complex-valued AM modulating signal  $m_k[n]$  of the first harmonic subband signal. The recovered AM modulating signal  $\hat{m}_k[n]$  is calculated either incoherently or coherently from  $\tilde{y}_k[n]$ , that is a similar procedure to the preceding calculation of  $m_k[n]$  from the unfiltered passband signal  $y_k[n]$ . A quantitative assessment is called an empirical modulation transfer function (MTF)  $G_e(\omega)$  as given on (45), where  $M_k(\omega)$  and  $\hat{M}_k(\omega)$  are the Fourier transforms of the original and the recovered AM modulating signal at channel k respectively [18, 19]. The modulation transfer function is to calculate the filter effectiveness from the AM-FM decomposition. Furthermore, the scenario examines the influence of original carrier as side information. In this case, the AM modulating  $\hat{w}_k[n]$  is calculated from demodulating  $\tilde{y}_k[n]$  by  $\tilde{c}_k[n]$  instead of  $\hat{c}_k[n]$ .

$$G_{e}(\omega) = \frac{1}{L} \sum_{k=1}^{L} \left| \frac{\hat{M}_{k}(\omega)}{M_{k}(\omega)} \right|$$
(45)



Figure 5. Complex-valued AM modulating signal of the first harmonic

Figure 6 shows the modulation transfer function for cases of with carrier side information to demodulate the passband signal. The linear phase LPF filter is constructed with 1-Hz cut-off frequency. The passband area of the MTF matches with the filter model while the stopband area is relatively constant. Thus, the proposed coherent detection method with carrier side information satisfies the band limiting condition of subband signal. Figure 7 shows the limited bandwidth of a subband signal of the 4<sup>th</sup> harmonic.



Figure 6. Modulation transfer function of the proposed coherent modulation transform with carrier side information



Figure 7. Normalized original and modulation filtered subband signal of the 4<sup>th</sup> harmonic

The recovered carrier estimation  $\hat{c}_k[n]$  differs from the original carrier estimation  $c_k[n]$  represented by the average difference between the estimated IFs as depicted on Figure 8 (a). The IF estimation of the proposed method can be implemented by finding the mean location of

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a distribution of energy in frequency . The continuous-time IF can be alternatively estimated with time-varying spectral Centre of Gravity (CoG) [18],

$$\varphi_k(t) = \frac{\int_{-\infty}^{\infty} \omega Y_k(\omega, t) d\omega}{\int_{-\infty}^{\infty} Y_k(\omega, t) d\omega}$$
(46)

, where the spectrogram of the passband signal with window f(t) is given by

$$Y_k(\omega,t) = \left| \int_{-\infty}^{\infty} f(\tau - t) y_k(\tau) e^{-j\omega\tau} d\tau \right|^2.$$
(47)

The average difference of IFs from CoG is given on Figure 8 (b). The IF estimation by the CoG is 3 dB better than the differential approximation of IPs in mean squared error. However, in terms of complexity the approximation by CoG is avoided since a new power spectrum is calculated for each incoming sample. In addition, the MTF of the proposed method with IF estimation by the differential approximation has better attenuation at stopband than the CoG.



Figure 8. The average difference between original and recovered Instantaneous Frequency (IF) of subband signals. (a) The IF is estimated from the differential approximation of Instantaneous Frequency (IP), while (b) is estimated from Centre of Gravity (CoG).

Furthermore, to check the result of the demodulation, the AM modulating signal  $m_k[n]$  is fed to the carrier estimator either by the proposed method  $\angle c_k[n] = \omega_k[n]$  on (38) or by the IF estimate of CoG  $\angle c_k[n] = \varphi_k[n]$  on (46). Both resulting IF estimates are centered at zero as shown on Figure 9, so that the demodulation process reflects the minimization of the bandwidth of the modulating signal  $m_k[n]$  [18].



Figure 9. The average Instantaneous Frequency (IF) of AM modulating signals. (a) The IF is estimated from the differential approximation of Instantaneous Frequency (IP), while (b) is estimated from Centre of Gravity (CoG).

#### C. Motion Artifact Reduction by LMS Adaptive Filter with Synthetic Noise Reference

Motion artifact reduction in PPG signals is implemented based on the methodology published in [17], except that the coherent modulation analysis, instead of FFT, SVD, or ICA,

#### Coherent Modulation Analysis of Photoplethysmographic Signals

was used to generate the synthetic noise reference. For the adaptive filter, we implemented the constant-step (CS) variation of the Normalized LMS algorithm in [20], as depicted on Figure. 10. The synthetic noise reference signal generation module is mainly aimed at generating estimated PPG spectrum, separated from that of the motion artifact noise. FFT-based bandpass filtering and SVD methods have been compared for this purpose. However, in this paper we introduce the utilization of coherent modulation analysis to generate the motion artifact-reduced PPG signal. The noise reference is generated from the AM modulating signals of harmonic components  $m_k[n]$  as on (48). Choosing  $L \neq 0$  will estimate the envelope of the PPG signal.

$$v_r[n] = \sum_{k=0}^{L} |m_k[n]|$$
(48)



Figure. 10. Motion artifact (MA) reduction by Constant Step Least Mean Square (CS-LMS) adaptive filter with synthetic noise reference generated by the proposed coherent modulation of harmonic components of PPG signal

Figure 11 describes that the raw 180-second PPG signal contains motion artifact noise at the second and third 60-second segments. The noise reference is generated from the 0<sup>th</sup> AM modulating signal. The filter convergences after 40 seconds and the artifact is reduced with constant step size  $\beta$  of 0.01 and 20-tap filter. More types of measurement conditions and the results of denoising are shown on Figure 12.



Figure 11. Motion artifact (MA) reduction for intermittently-moved finger in vertical direction. The first, second, and third 60-second segments resulted from finger in relaxed condition, vertically moved every 20 seconds, and every 10 seconds, respectively.

### 4. Conclusions

By allowing the filterbank track the Instantaneous Frequency (IF) of each harmonic component, the FM modulated signals are estimated coherently with the demodulation of the AM modulating signals. Thus, the filterbank acts as a time-varying comb filter that passes the frequency band around the harmonic components. We have compared the IF estimations by differential approximation of Instantaneous Phase (IP) and by the Centre of Gravity (CoG). In spite of complexity burden, the CoG IF estimation empirically offers 3-dB improvement in mean squared error between the original and recovered IF of subband signals than by the differential approximation of IP. A quantitative measurement is reflected by the modulation projection test in which the Modulation Transfer Function (MTF) of the proposed method resembles the filter model at passband frequency and has relatively constant magnitude at stopband frequency. The decomposition of the harmonics signal into AM and FM modulating signals, therefore, requires only limited number of filters to construct the filterbank, depending on the number of harmonic components is needed for the analysis, e.g. in our PPG signals we are able to sufficiently reconstruct the time domain signal from only 6 harmonic components. We also have shown graphically the effectiveness of applications of our methods to reduce different motion artifacts. Finally, the proposed method in this paper is useful not only for the photoplethysmographic (PPG) signal, but also for harmonic multimodular signals where the unimodular instantaneous frequency of each harmonic component visibly changes in timefrequency domain.



Figure 12. Motion artifact (MA) reduction from (a) deep-fast-hold-normal breathings, (b) intermittently-lifted arm, (c) intermittently-pressed finger, (d) induced reactive hyperemia, and (e) intermittently-cuffed arm

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**Richard Mengko** obtained the B.Eng. degree in electrical engineering from the Institut Teknologi Bandung (ITB), Bandung, Indonesia, in 1978. He studied digital image processing and pattern recognition for the Ph.D. degree from ENSERG-INP de Grenoble, France, in 1985. He is now an Associate Professor with the School of Electrical Engineering and Informatics, ITB. His previous research topics were text-to-speech for Indonesian language, spectral analysis, WiMAX, and smart card system. The main field of his current research interests includes instrumentations and signal analysis

especially for biomedical applications. Ones of his research outcomes are the 12-lead ECG, the device of non-invasive vascular analyzer, and the EEG system for intraoperative monitoring. He is also among who initiated Biomedical Engineering in ITB.



**Habibur Muhaimin** graduated from the Institut Teknologi Bandung (ITB), Bandung, Indonesia, majoring in telecommunication engineering in 2010 and from the University of Manchester, Manchester, the UK, in digital image and signal processing in 2012, for the B.Eng. and M.Sc. degrees, respectively. From 2013 to 2014, he was a Research Assistant with the Telecommunication Engineering Research Group, ITB. In 2015, he started his academic career as a Lecturer at the School of Electrical Engineering and Informatics, ITB. His research interests are signal processing-related topics,

which include applications of signal processing to radar and hearing.



**Tati Latifah R. Mengko** received the B.Eng. degree in electrical engineering from the Institut Teknologi Bandung (ITB), Bandung, Indonesia, in 1977. In 1985, she obtained the Ph.D. degree from the Institut National Polytechnique de Grenoble at École Nationale Supérieure d'Électronique et de Radioélectricité de Grenoble (ENSERG), France, where she studied about texture-based image processing. Since 2005, she has been a Professor with the School of Electrical Engineering and Informatics, ITB. In 2015, she was granted an innovation award from ITB due to her contribution to the

development of the device of non-invasive vascular analyzer. She has chaired numerous conferences including International Conference on Instrumentation, Communication, Information Technology, and Biomedical Engineering (ICICI-BME). She is currently the Head of Biomedical Engineering Research Group, ITB. Her research interests are telemedicine and biomedical signal processing.