



## An Intelligent Water Drop Algorithm for Solving Optimal Reactive Power Dispatch Problem

K. Lenin and M. Surya Kalavathi

Department of electrical & electronics engineering  
Jawaharlal Nehru Technological University Hyderabad  
Kukatpally, Hyderabad - 500 085, Andhra Pradesh, India  
gklenin@gmail.com

**Abstract:** This paper presents an algorithm for solving the multi-objective reactive power dispatch problem in a power system. Modal analysis of the system is used for static voltage stability assessment. Loss minimization and maximization of voltage stability margin are taken as the objectives. Generator terminal voltages, reactive power generation of the capacitor banks and tap changing transformer setting are taken as the optimization variables. In this paper an intelligent water drop (IWD) algorithm has been proposed to solve this combinatorial optimization problem. Intelligent water drop algorithm is a swarm-based nature inspired optimization algorithm, which has been inspired from natural rivers. A natural river often finds good paths among lots of possible paths in its ways from source to destination and finally find almost optimal path to their destination. These ideas are embedded into proposed algorithm for solving reactive dispatch problem.

**Index Terms:** Modal analysis, optimal reactive power, Transmission loss, Optimization, Intelligent Water Drop Algorithm.

### 1. Introduction

Optimal reactive power dispatch problem is one of the difficult optimization problems in power systems. The sources of the reactive power are the generators, synchronous condensers, capacitors, static compensators and tap changing transformers. The problem that has to be solved in a reactive power optimization is to determine the required reactive generation at Various locations so as to optimize the objective function. Here the reactive power dispatch problem involves best utilization of the existing generator bus voltage magnitudes, transformer tap setting and the output of reactive power sources so as to minimize the loss and to enhance the voltage stability of the system. It involves a non linear optimization problem. Various mathematical techniques have been adopted to solve this optimal reactive power dispatch problem. These include the gradient method [1-2], Newton method [3] and linear programming [4-7]. The gradient and Newton methods suffer from the difficulty in handling inequality constraints. To apply linear programming, the input- output function is to be expressed as a set of linear functions which may lead to loss of accuracy. Recently global Optimization techniques such as genetic algorithms have been proposed to solve the reactive power flow problem [8, 9].

In this paper, a new approach intelligent water drop (IWD) algorithm [10], is used to solve the voltage constraint reactive power dispatch problem, the proposed algorithm identify the optimal values of generation bus voltage magnitudes, transformer tap setting and the output of the reactive power sources as to minimize the transmission loss to improve the voltage stability. The effectiveness of the proposed approach is demonstrated through IEEE-30 bus

system. The test results show the proposed algorithm gives better results with less computational burden and is fairly consistent in reaching the near optimal solution

In recent years, the problem of voltage stability and voltage collapse has become a major concern in power system planning and operation. To enhance the voltage stability, voltage magnitudes alone will not be a reliable indicator of how far an operating point is from the collapse point [11]. The reactive power support and voltage problems are intrinsically related. Hence, this paper formulates the reactive power dispatch as a multi-objective optimization problem with loss minimization and maximization of static voltage stability margin (SVSM) as the objectives. Voltage stability evaluation using modal analysis [12] is used as the indicator of voltage stability.

## 2. Voltage Stability Evaluation

### A. Modal analysis for voltage stability evaluation

Modal analysis is one of the methods for voltage stability enhancement in power systems. In this method, voltage stability analysis is done by computing eigen values and right and left eigen vectors of a jacobian matrix. It identifies the critical areas of voltage stability and provides information about the best actions to be taken for the improvement of system stability enhancements. The linearized steady state system power flow equations are given by.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} \quad (1)$$

where

$\Delta P$  = Incremental change in bus real power.

$\Delta Q$  = Incremental change in bus reactive Power injection

$\Delta \theta$  = incremental change in bus voltage angle.

$\Delta V$  = Incremental change in bus voltage Magnitude

$J_{P\theta}$ ,  $J_{PV}$ ,  $J_{Q\theta}$ ,  $J_{QV}$  jacobian matrix are the sub-matrixes of the System voltage stability is affected by both P and Q. However at each operating point we keep P constant and evaluate voltage stability by considering incremental relationship between Q and V.

To reduce (1), let  $\Delta P=0$ , then.

$$\Delta Q = [J_{QV} - J_{Q\theta} J_{P\theta}^{-1} J_{PV}] \Delta V = J_R \Delta V \quad (2)$$

$$\Delta V = J^{-1} \Delta Q \quad (3)$$

Where

$$J_R = (J_{QV} - J_{Q\theta} J_{P\theta}^{-1} J_{PV}) \quad (4)$$

$J_R$  is called the reduced Jacobian matrix of the system.

### B. Modes of Voltage instability:

Voltage Stability characteristics of the system can be identified by computing the eigen values and eigen vectors, Let

$$J_R = \xi \Lambda \eta \quad (5)$$

Where,

$\xi$  = right eigenvector matrix of  $J_R$

$\eta$  = left eigenvector matrix of JR  
 = diagonal eigenvalue matrix of JR and

$$J_R^{-1} = \xi \Lambda^{-1} \eta \quad (6)$$

From (3) and (6), we have

$$\Delta V = \xi^{-1} \eta \Delta Q \quad (7)$$

Or

$$\Delta V = \sum_i \frac{\xi_i \eta_i}{\lambda_i} \Delta Q \quad (8)$$

where  $\xi_i$  is the  $i^{\text{th}}$  column right eigenvector and  $\eta$  the  $i^{\text{th}}$  row left eigenvector of JR.  
 $\lambda_i$  is the  $i^{\text{th}}$  eigen value of JR.

The  $i^{\text{th}}$  modal reactive power variation is,

$$\Delta Q_{mi} = K_i \xi_i \quad (9)$$

where,

$$K_i = \sum_j \xi_{ij}^2 - 1 \quad (10)$$

Where,

$\xi_{ji}$  is the  $j^{\text{th}}$  element of  $\xi_i$

The corresponding  $i^{\text{th}}$  modal voltage variation is

$$\Delta V_{mi} = [1 / \lambda_i] \Delta Q_{mi} \quad (11)$$

It is seen that, when the reactive power variation is along the direction of  $\xi_i$  the corresponding voltage variation is also along the same direction and magnitude is amplified by a factor which is equal to the magnitude of the inverse of the  $i^{\text{th}}$  eigenvalue. In this sense, the magnitude of each eigenvalue  $\lambda_i$  determines the weakness of the corresponding modal voltage. The smaller the magnitude of  $\lambda_i$ , the weaker will be the corresponding modal voltage. If  $|\lambda_i| = 0$  the  $i^{\text{th}}$  modal voltage will collapse because any change in that modal reactive power will cause infinite modal voltage variation.

In (8), let  $\Delta Q = e_k$  where  $e_k$  has all its elements zero except the  $k^{\text{th}}$  one being 1. Then,

$$\Delta V = \sum_i \frac{\eta_{ik} \xi_i}{\lambda_i}$$

where  $\eta_{ik}$  the  $k^{\text{th}}$  element of  $\eta_i$ .

(12)

V - Q sensitivity at bus k,

$$\frac{\partial V_k}{\partial Q_k} = \sum_i \frac{\xi_{ki} \eta_{ik}}{\lambda_i} = \sum_i \frac{P_{ki}}{\lambda_i} \quad (13)$$

A system is voltage stable if the eigenvalues of the Jacobian are all positive. Thus the results for voltage stability enhancement using modal analysis for the reduced jacobian matrix is when

- eigen values  $\lambda_i > 0$ , the system is under stable condition
- eigen values  $\lambda_i < 0$ , the system is unstable
- eigen values  $\lambda_i = 0$ , the system is critical and collapse state occurs

### 3. Problem Formulation

The optimal reactive power dispatch problem is formulated as an optimization problem in which a specific objective function is minimized while satisfying a number of equality and inequality constraints. The objectives of the reactive power dispatch problem considered here is to minimize the system real power loss and maximize the static voltage stability margins (SVSM). This objective is achieved by proper adjustment of reactive power variables like generator voltage magnitude ( $V_{gi}$ ), reactive power generation of capacitor bank ( $Q_{ci}$ ), and transformer tap setting ( $tk$ ). Power flow equations are the equality constraints of the problems, while the inequality constraints include the limits on real and reactive power generation, bus voltage magnitudes, transformer tap positions and line flows. This objective function is subjected to the following constraints:

#### A. Real power losses:

To minimize the real power loss in the system, this can be expressed as

$$\text{Minimize } P_{Loss} = \sum_{\substack{k \in N \\ k=(i,j)}} g_k (V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij}) \quad (14)$$

#### B. Maximize SVSM:

This is the most widely accepted index for proximity of voltage collapse. It is defined as the largest load change that the power system may sustain at a bus or collective of buses from a well defined operating point. (Base case) Using the modal analysis the minimal eigen value of the non-singular power flow jacobian matrix has been used to find the maximum static voltage stability margin in this proposed approach.

#### C. Equality Constraints

These constraints represent the typical load flow equation such as

$$P_i - V_i \sum_{j=1}^{N_B} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0, i \in N_B - 1 \quad (15)$$

$$Q_i - V_i \sum_{j=1}^{N_B} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0, i \in N_{PQ} \quad (16)$$

#### D. Inequality Constraints

These constraints represent the system operating constraints. Generator bus voltages ( $V_{gi}$ ), reactive power generated by the capacitor ( $Q_{ci}$ ), transformer tap setting ( $tk$ ), are control variables and they are self restricted. Load bus voltages ( $V_{load}$ ) reactive power generation of generator ( $Q_{gi}$ ) and line flow limit ( $Sl$ ) are state variables, whose limits are satisfied by adding a penalty terms in the objective function. These constraints are formulated as

##### (i) Voltage limits

$$V_i^{\min} \leq V_i \leq V_i^{\max}; i \in N_B \quad (17)$$

##### (ii) Generator reactive power capability limit

$$Q_{gi}^{\min} \leq Q_{gi} \leq Q_{gi}^{\max}; i \in N_g \quad (18)$$



available paths. The lower the soil of the path, the more chance it has for being selected by the IWD.

### B. Intelligent Water Drops Algorithm

The IWD algorithm gets a representation of the problem in the form of a graph (N, E) with the node set N and edge set E. Then, each IWD begins constructing its solution gradually by traveling on the nodes of the graph along the edges of the graph until the IWD finally completes its solution. One iteration of the algorithm is complete when all IWDs have completed their solutions. After each iteration, the iteration best solution  $T^{IB}$  is found and it is used to update the total best solution  $T^{TB}$ . The amount of soil on the edges of the iteration-best solution  $T^{IB}$  is reduced based on the goodness (quality) of the solution. Then, the algorithm begins another iteration with new IWDs but with the same soils on the paths of the graph and the whole process is repeated. The algorithm stops when it reaches the maximum number of iterations  $iter_{max}$  or the total-best solution  $T^{TB}$  reaches the expected quality. The IWD algorithm has two kinds of parameters. One kind is those that remain constant during the lifetime of the algorithm and they are called ‘static parameters’. The other kind is those parameters of the algorithm, which are dynamic and they are reinitialized after each iteration of the algorithm.

*The algorithm of IWD is specified in the following steps:*

1. The graph (N, E) of the problem is given to the algorithm. The quality of the total-best solution  $T^{TB}$  is initially set to the worst value:  $q(TTB) = \infty$ . The maximum number of iterations  $iter_{max}$  is specified by the user. The iteration count  $iter_{count}$  is set to zero. The number of water drops NIWD is set to a positive integer value, which is usually set to the number of nodes Nc of the graph. For velocity updating, the parameters are  $av = 1$ ,  $bv = 0.01$  and  $cv = 1$ . For soil updating,  $as = 1$ ,  $bs = 0.01$  and  $cs = 1$ . The local soil updating parameter  $\rho_n = 0.9$ , which is a small positive number less than one. The global soil updating parameter  $\rho_{IWD} = 0.9$ , which is chosen from [0, 1]. Moreover, the initial soil on each path (edge) is denoted by the constant *InitSoil* such that the soil of the path between every two nodes  $i$  and  $j$  is set by  $soil(i, j) = \text{InitSoil}$ . The initial velocity of each IWD is set to  $\text{InitV}_{el}$ . Both parameters *InitSoil* and  $\text{InitV}_{el}$  are user selected and they should be tuned experimentally for the application.
2. Every IWD has a visited node list  $V_c(\text{IWD})$ , which is initially empty:  $V_c(\text{IWD}) = \emptyset$ . Each IWDs velocity is set to  $\text{InitV}_{el}$ . All IWDs are set to have zero amount of soil.
3. Spread the IWDs randomly on the nodes of the graph as their first visited nodes.
4. Update the visited node list of each IWD to include the nodes just visited.
5. Repeat Steps 5.1 to 5.4 for those IWDs with partial solutions.

*A. For the IWD residing in node  $i$ , choose the next node  $j$ , which does not violate any constraints of the problem and is not in the visited node list  $V_c(\text{IWD})$  of the IWD, using the following probability*

$$p_i^{\text{IWD}}(j):$$

$$p_i^{\text{IWD}}(j) = \frac{f(soil(i, j))}{\sum_{k \notin V_c(\text{IWD})} f(soil(i, k))}$$

(22)

such that

$$f(soil(i, j)) = \frac{1}{\epsilon_s + g(soil(i, j))}$$



$$T^{TB} = \begin{cases} T^{TB} & q(T^{TB}) \geq q(T^{IB}), \\ T^{IB} & otherwise \end{cases} \quad (31)$$

9. Increment the iteration number by  $Iter_{count} = Iter_{count} + 1$ . Then, go to Step 2 if  $Iter_{count} < Iter_{max}$ .

10. The algorithm stops here with the total-best solution  $T^{TB}$ .

And the flow chart of the proposed IWD algorithm which has been applied for reactive dispatch problem given the figure 1.

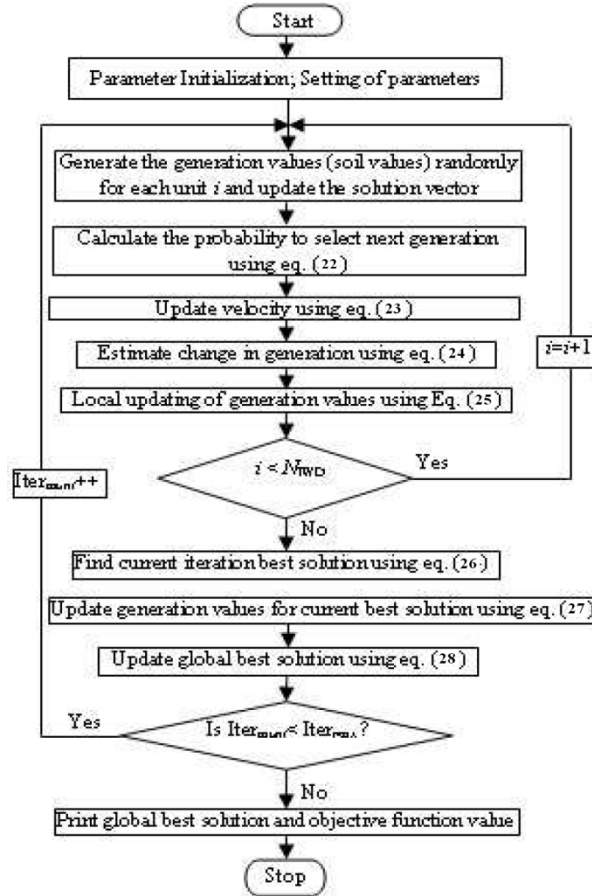


Figure 1. Flowchart of the proposed IWD algorithm

### C. Formation of the fitness function

In the optimal reactive power dispatch problem, the objective is to minimize the total real power loss while satisfying the constraints (14) to (21). For each individual, the equality constraints are satisfied by running Newton-Raphson algorithm and the constraints on the state variables are taken into consideration by adding penalty function to the objective function. With the inclusion of the penalty factors, the new objective function then becomes,

$$Min F = P_{loss} + w E_{g_{max}} + \sum_{i=1}^{N_{PQ}} Y P_i + \sum_{j=1}^{N_V} Q P_j + \sum_{l=1}^{N_I} L P_l \quad (31)$$





Table 1. Results of IWD – RPD optimal control variables

Control variables	Variable setting
V1	1.048
V2	1.046
V5	1.044
V8	1.035
V11	1.012
V13	1.042
T11	1.09
T12	1.02
T15	1.1
T36	1.0
Qc10	3
Qc12	2
Qc15	4
Qc17	0
Qc20	3
Qc23	4
Qc24	3
Qc29	3
Real power loss	4.4825
SVSM	0.2362



Table 4. Limit Violation Checking of State Variables

State variables	limits		ORPD	VSCRPD
	Lower	upper		
Q1	-20	152	1.3422	-1.3269
Q2	-20	61	8.9900	9.8232
Q5	-15	49.92	25.920	26.001
Q8	-10	63.52	38.8200	40.802
Q11	-15	42	2.9300	5.002
Q13	-15	48	8.1025	6.033
V3	0.95	1.05	1.0372	1.0392
V4	0.95	1.05	1.0307	1.0328
V6	0.95	1.05	1.0282	1.0298
V7	0.95	1.05	1.0101	1.0152
V9	0.95	1.05	1.0462	1.0412
V10	0.95	1.05	1.0482	1.0498
V12	0.95	1.05	1.0400	1.0466
V14	0.95	1.05	1.0474	1.0443
V15	0.95	1.05	1.0457	1.0413
V16	0.95	1.05	1.0426	1.0405
V17	0.95	1.05	1.0382	1.0396
V18	0.95	1.05	1.0392	1.0400
V19	0.95	1.05	1.0381	1.0394
V20	0.95	1.05	1.0112	1.0194
V21	0.95	1.05	1.0435	1.0243
V22	0.95	1.05	1.0448	1.0396
V23	0.95	1.05	1.0472	1.0372
V24	0.95	1.05	1.0484	1.0372
V25	0.95	1.05	1.0142	1.0192
V26	0.95	1.05	1.0494	1.0422
V27	0.95	1.05	1.0472	1.0452
V28	0.95	1.05	1.0243	1.0283
V29	0.95	1.05	1.0439	1.0419
V30	0.95	1.05	1.0418	1.0397

Table 5. Comparison of Real Power Loss

Method	Minimum loss
Evolutionary programming[13]	5.0159
Genetic algorithm[14]	4.665
Real coded GA with Lindex as SVSM[15]	4.568
Real coded genetic algorithm[16]	4.5015
Proposed IWD method	4.4825



- [11] C.A. Canizares , A.C.Z.de Souza and V.H. Quintana , “ Comparison of performance indices for detection of proximity to voltage collapse ,” *IEEE Transactions on power system vol. 11. no.3 , pp.1441-1450, Aug 1996*
- [12] B.Gao, G.K Morison P. Kundur “voltage stability evaluation using modal analysis“ *IEEE Transactions on Power Systems ,Vol 7, No .4 , November 1992.*
- [13] Wu Q H, Ma J T. Power system optimal reactive power dispatch using evolutionary programming. *IEEE Transactions on power systems 1995; 10(3): 1243-1248*
- [14] S.Durairaj, D.Devaraj, P.S.Kannan ,” Genetic algorithm applications to optimal reactive power dispatch with voltage stability enhancement” , *IE(I) Journal-EL Vol 87,September 2006.*
- [15] D.Devaraj ,”Improved genetic algorithm for multi – objective reactive power dispatch problem” *European Transactions on electrical power 2007 ; 17: 569-581*
- [16] P. Aruna Jeyanthi and Dr. D. Devaraj “Optimal Reactive Power Dispatch for Voltage Stability Enhancement Using Real Coded Genetic Algorithm” *International Journal of Computer and Electrical Engineering, Vol. 2, No. 4, August, 2010 1793-8163.*



**K. Lenin** has received his B.E., Degree, electrical and electronics engineering in 1999 from university of madras, Chennai, India and M.E., Degree in power systems in 2000 from Annamalai University, Tamil Nadu, India. At present pursuing Ph.D., degree at JNTU, Hyderabad, India.



**M. Surya Kalavathi** has received her B.Tech. Electrical and Electronics Engineering from SVU, Andhra Pradesh, India and M.Tech, power system operation and control from SVU, Andhra Pradesh, India. she received her Phd. Degree from JNTU, hyderabad and Post doc. From CMU – USA. Currently she is Professor and Head of the electrical and electronics engineering department in JNTU, Hyderabad, India and she has Published 16 Research Papers and presently guiding 5 Ph.D. Scholars. She has specialised in Power Systems, High Voltage Engineering and Control Systems. Her research interests include Simulation studies on Transients of different power system equipment. She has 18 years of experience. She has invited for various lectures in institutes.