

Reconstruction of Holographic Microscopy Images Based on Matching Pursuits on A Pair of Domains

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Abstract: We propose a new reconstruction method of an object image from its digital hologram. The proposed matching pursuit on a pair of domains (MPPD) method employs spatial-domain bases and their (Fresnel) transform-domain pair. The transform domain bases are used to decompose the hologram, which yield a set of coefficients. Then, these coefficients are used to reconstruct the spatial-domain object image using the predefined spatial bases. We show the robustness of the proposed method against noise on a simulated hologram of spherical particles. By employing spatial-domain Gaussian bases and its transform pair, the image of these particles are recovered successfully. A possibility to extend the 2D (two-dimensional) case to a 3D (three-dimensional) one for slice reconstruction from a single hologram is also explored. The effectiveness of the proposed method is demonstrated by using a real microscopic-hologram of silica gel spherical particles, which shows promising results.

1. Introduction

Holography has been popularly known as an imaging technique that is capable of recording and reconstructing a 3D (three-dimensional) image by employing a laser. However, the first holographic device constructed by Gabor, was not intended for this purpose, but for correcting aberrations of an electron microscope by recording the interference of object wave with a reference one [1]-[3]. Leith and Upatnieks proposed significant improvements by proposing an off-axis hologaphic imaging method [4], which capable to separate a real image from its virtual and zero order ones. They also realized the relationship between holography and well-known signal processing techniques widely used in communication, i.e., the modulation, frequency dispersion, and square-law detection.

A digital holographic imaging (DHI) system is constructed by replacing the film in the analog holographic imaging (AHI) with a digital camera. Based on the digitally recorded hologram, the object image is then reconstructed numerically. Considering its central role, the numerical reconstruction of the object image is one of the most important issues in the DHI.

Previously, a digital holographic microscope imaging system capable to record and reconstruct images of microscopic organisms has been implemented by researchers [5], [6]. Various kinds of inline digital holographic imaging techniques have also been developed. In [7], the authors presented tile-superposition technique for in-line digital holography, which can be the basis of high-resolution wide-field imaging by multispot illumination with NA (Numerical Aperture) of 0.7. A single shot high-resolution imaging using partially coherent laser light illumination achieving NA of 0.8 has also been introduced in [8], where rearranging sample carrier enables one to get a hologram that is free from disturbing interference. The capability of imaging actual objects, among others are E-Coli bacteria, HeLa cells, and Fibroblast cells, have also been demonstrated in [9]. In principle, these devices will also capable to record and play a 3D (three-dimensional) movies of observed living microorganisms. Current progress on the DHI research was reported by Schnars and Juptner [10].

Actually, compared to other microscopic imaging modalities, such as the confocal and electron microscopy, holography technique offers a few more advantages; among others are: non-invasiveness, high-speed 3D capability compared to scan-mode imaging, no vacuum requirement, and its capability to work in the presence of ambient light [5].

The DHI is not only applicable to visible light, but also works with other kind of waves as well, which actually was demonstrated by Gabor in his electron-wave holographic microscope. A soft X-ray from synchrotron radiation has been used by Barth [11] to image microscopic object with 0.37 μm resolution. Using electron-wave, Tonomura et al. [12] used holography to demonstrate that electromagnetic vector potential is a physical object, rather than a mere mathematical concept, by demonstrating the Aharonov-Bohm effect. This experiment also shows that holography is a useful technique in fundamental research.

Considering the wide range applicability of the DHI, numerical reconstruction will play an important role. In developing a reconstruction algorithm, high quality reconstruction results with fewer artifacts and distortions are desired. In this paper, we propose a new method of matching pursuit on a pair of domains (MPPD) to address the image reconstruction problems. We evaluate the performance of the MPPD for a simulated and an actual digital holograms. We also investigated a possibility to use the MPPD to reconstruct slice-images in a 3D imaging scheme.

The usage of two-domains and the iterative algorithm to solve the reconstruction problem have been addressed by researchers, although in a different settings. Rabadi et al. proposed an iterative algorithm that employs multiresolution pyramid to reconstruct an image from the magnitude of Fourier transform [13]. The method also used two-domains, i.e., the object's and the Fourier's domains. In contrast to this method where the two-domain is used to implement constraint of the algorithm, we have used it for a different purpose. Each of our domains consists of bases functions, i.e., the localized spatial bases and their transform domain pair. The weights are estimated from the transform-domain bases, whereas the reconstruction is conducted in the spatial bases. In addition, Jesacher et al. proposed a method that also employs iterative phase-retrieval algorithm that capable to suppress twin image [14]. Although both of this method and ours used iterative technique, they are very different in nature. In ours, the twin image has been filtered out in the demodulation process so that there is no such issue when the iterative reconstruction algorithm is performed. Issues related to twin image and phase recovery also discussed by Cuche et al. [15] and Yamaguchi and Zhang [16].

The rest of this paper is organized as follows. Section 2 explains the basic theory of hologram formation, numerical reconstruction, and description of the proposed method. The comparison between the proposed methods with the direct inverse transform for both a simulated a Gaussian particle and an actually measured hologram is described in Section 3. This paper is concluded in Section 4.

2. Materials and Methods

A. Imaging Geometry and Mathematical Model

Consider a generic holographic imaging system. Let $o(\xi,\eta) = |o(\xi,\eta)| \exp j\varphi_o(\xi,\eta)$ be the wave scattered by an object and $r(\xi,\eta) = |r(\xi,\eta)| \exp j\varphi_r(\xi,\eta)$ be the reference wave arriving on a screen located at a particular distance from the object, where both of these waves interferes. The intensity distribution on the screen is $I(\xi,\eta) = |o(\xi,\eta) + r(\xi,\eta)|^2$, or

$$I(\xi,\eta) = |o(\xi,\eta)^{2}| + |r(\xi,\eta)|^{2} + r^{*}(\xi,\eta)o(\xi,\eta) + r(\xi,\eta)o^{*}(\xi,\eta)$$
(1)

where $(...)^*$ denotes complex conjugate. In the analog holography, the intensity is recorded on a film that yields a hologram whose transmittance distribution is

$$h(\xi,\eta) = h_0 + \beta \tau I(\xi,\eta) \tag{2}$$

where β is a constant, τ is the exposure time, and h_0 is the amplitude of transmission of the unexposed plate [10].

Reconstruction in the analog holography is conducted by illuminating the hologram with a reference wave $r(\xi, \eta)$, i.e.,

$$r(\xi,\eta)h(\xi,\eta) = \left(h_0 + \beta\tau \left(\left|o(\xi,\eta)\right|^2 + \left|r(\xi,\eta y)\right|^2\right)\right)r(\xi,\eta) + \beta\tau \left|r(\xi,\eta)\right|^2 o(\xi,\eta) + \beta\tau r^2(\xi,\eta)o^*(\xi,\eta)$$
(3)

In the r.h.s. (right hand side) of the equation (3), the first term represents the reference wave (multiplied by a constant), the second term is a reconstructed object wave which will be observed as an object image, and the third one is a conjugate image. In the inline (holography) imaging, the object and conjugate terms cannot be separated because their spatial spectrum are overlapping. On the other hand, these terms can be separated easily in the off-axis hologaphic imaging by demodulation.

The playback in an AHI to show the object image is conducted by illuminating the hologram by a coherent light, such as a laser. On the other hand, the film is replaced by a digital camera in the DHI system. To show the object image, a computer is required to calculate and reconstruct it based on a recorded digital hologram. The following section describes how such a process is conducted by a numerical algorithm.

B. Numerical Reconstruction

One of the most important issues in the digital holography is numerical reconstruction of the object image from the recorded hologram. Usually, the calculation is based on the Fresnel approximation of the Fresnel-Kirchhoff integral that relates a wave $\Psi(\xi,\eta) = r(\xi,\eta)h(\xi,\eta)$, which is located at *z*=0, with the object's diffracted wave $\Phi(x,y)$, which is reconstructed at *z*=*d*, as follows [19]

$$\Phi(x,y) = \frac{\exp(jkd)}{j\lambda d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Psi(\xi,\eta) \exp\left\{-j\frac{k}{2d}\left((x-\xi)^2 + (y-\eta)^2\right)\right\} d\xi \, d\eta \tag{4}$$

In the DHI, we can assume $h_0=0$ in (2) and, if the reference is a plane wave, we can select it to possess a real amplitude *r*. Equation (4) can also be expressed as a convolution integral between $\Psi(\xi, \eta)$ with a kernel g(x, y), which is defined as

$$g(x, y) = -j \exp\left(jkz\right) \left\{ \frac{1}{\tau^2} \exp j\pi \left(\frac{\|\vec{x}\|}{\tau}\right)^2 \right\}$$
(5)

In equation (5), $\tau = \sqrt{\lambda z}$, $k = 2\pi/\lambda$, and $\|\vec{x}\| = \sqrt{x^2 + y^2}$. Therefore, we can rewrite the convolution integral in (4) as

$$\Phi(x, y) = -je^{jkd} \{\Psi * K_{\tau}\}(x, y)$$
(6)

Where

$$K_{\tau}(x, y) \equiv K_{\tau}(\vec{x}) = \frac{1}{\tau^2} \exp j\pi \left(\frac{\|\vec{x}\|}{\tau}\right)^2$$
(7)

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Following convention in [19], the term inside the curly bracket {.} in (6) will be defined as the Fresnel transform of Ψ and it is denoted by $\tilde{\Psi}_r(x, y)$.

We can now summarize that the wave $\Psi(\xi,\eta)$ representing the demodulated hologram $r(\xi,\eta)h(\xi,\eta)$, is related to the diffracted wave $\Phi(x,y)$, i.e. the reconstructed object image, through the following Fresnel transforms

$$\Phi(x, y) = -je^{jkd} \widetilde{\Psi}_{r}(x, y)$$
(8)

In principle, the numerical reconstruction of the hologram is based on (8), i.e., given discrete values of the hologram, we calculate $\Phi(x, y)$ by using inverse Fresnel transform. Due to imperfection of measurements and noise, in reality, we only get an approximate value that will be denoted by $\hat{\Phi}(x, y)$.

In the DHI, the object image is reconstructed from a digitized hologram. For a hologram that is sampled on an $N \times N$ rectangular grid with steps $\Delta \xi$ and $\Delta \eta$ along the coordinates, the discrete Fresnel transform is

$$\Phi(m,n) = \frac{j}{\lambda d} \exp\left[j\pi\lambda d\left(\frac{m^2}{N^2 \Delta \xi^2} + \frac{n^2}{N^2 \Delta \eta^2}\right) \right] \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} \Psi(k,l) \exp\left[-j\frac{\pi}{\lambda d} \left(k^2 \Delta x^2 + l^2 \Delta y^2\right) \right]$$
(9)

$$\times \exp\left[j2\pi \left(\frac{km}{N} + \frac{ln}{N}\right) \right]$$

The phase factor in front of the summation can be neglected for most practical applications. Since the summation in (9) is an expression of two-dimensional Fourier transform, the object image can be calculated by multiplying the hologram with $\exp\left[\left(-j\pi/\lambda d\right)\left(k^2\Delta x^2 + l^2\Delta y^2\right)\right]$, which is followed by applying the inverse discrete Fourier transform to the product. Then, the reconstruction of the object image can be expressed in a simple form as

$$\hat{\Phi} = FFT^{-1} \{ FFT \{ h \cdot r \} \cdot FFT \{ g \} \}$$
(10)

where $\hat{\Phi}$, h, r, and g is the discrete version of the same notation explained before. The usage of FFT in (10) and other version of fast-transforms described in [17] and [18], enables fast reconstruction process. Object image reconstruction that is performed based on equation (10) will be referred to as the direct inversion method's image.

C. Matching Pursuits on A Pair of Domains

The proposed MPPD method uses localized bases in the spatial domain, based on which its Fresnel-domain's pairs are constructed. The hologram, which is a function in the Fresnel domain, is decomposed by an iterative matching-and-substraction process with the Fresnel-domain bases. Then, the corresponding spatial bases are properly chosen and weighted to reconstruct the object's image.

The hologram-modulated wave $\Psi(\xi, \eta)$ that is diffracted wave originated from z = 0 and the wave $\Phi(x, y)$ which arrives in location z=d that represents the reconstructed object image, are related by the Fresnel transform. This relationship can be expressed by [19]

$$\left[\Phi * K_{\tau}^{-1}\right](\xi,\eta) = \Psi(\xi,\eta) \leftrightarrow \Phi(x,y) = \left[\Psi * K_{\tau}\right](x,y)$$
(11)

where $K_{\tau}^{-1} = (1/\tau^2) e^{-j\pi(||x||/\zeta)^2}$.

We assume that the object image can be linearly decomposed into a number of twodimensional spatially-localized functions $\phi_i(x, y)$, i.e.,

$$\Phi(x, y) = \sum_{k=1}^{K} a_k \phi_k(x, y)$$
(12)

Applying Fresnel transform $F_r[.]$ to both sides of (12) and considering the linearity property of this identity, we arrive to

$$F_{r}[\Phi(x, y)] = \sum_{k=1}^{K} a_{k} F_{r}[\phi_{k}(x, y)]$$
(13)

or equivalently

$$\Psi(\xi,\eta) = \sum_{k=1}^{K} a_k \psi_k(\xi,\eta) \tag{14}$$

where $\psi_k(\xi,\eta)$ is the Fresnel transform of the spatial domain bases $\phi_k(x, y)$. Since (12) and (14) express linear combination of bases functions, both of the weights or coefficients a_k in (12) and (14) are identical. The last results expressed in (14) states that the hologram Ψ is a linear combination of Fresnel-transform of the spatial-function $\psi_k(x, y)$.

In our reconstruction problem, the hologram is the known parameter, based on which the coefficients a_k are estimated. We reconstruct the object image in (12) by using estimated coefficients and the pre-defined bases $\phi_k(x,y)$. This reconstruction method is formulated into the MPPD algorithm shown in the following pseudocode.

Matching Pursuits on a Pair of Domains (MPPD) Algorithm

- Initializations
 - Construct localized spatial bases { $\phi_k(x, y)$ } and its pair { $\psi_k(\xi, \eta) = F_k[\phi_k(x, y)]$ }.
 - Retrieve an estimate of Fresnel-domain object $\Psi(\xi, \eta)$ from the hologram $h(\xi, \eta)$.
 - Initialize iteration index *m*=1
 - Initialize residual hologram $\rho_m(\xi, \eta) = \Psi(\xi, \eta)$
 - Initialize estimated image $\hat{\Phi}_m(x, y) = 0$.
 - Define initial residual energy E_0 as a big positive number.
- MPPD Iterations
 - WHILE $E_m > E_{m-1}$
 - Perform projection of residual hologram ρ_m(ξ, η) into Fresnel-transformed Gaussian bases {ψ_k} and select the maximum values a_k
 a_k = max {⟨ψ_k, ρ_m⟩}
 - Image reconstruction:
 - Use a_k to reconstruct the image

$$\Phi_{m+1}(x, y) = \Phi_m(x, y) + a_k \phi_k(x, y)$$

• Reduce the residual hologram with corresponding value

$$\rho_{m+1}(\xi,\eta) = \rho_m(\xi,\eta) - a_k \psi_k(\xi,\eta)$$

• Increment the iteration index m

END

3. Results and Discussion

A. Processing Flows: Synthesis and Analysis of a Digital Off-Axis Hologram

There are two kinds of hologram data used in the experiments, i.e., a simulated and an actually measured holograms. Both of them assume an off-axis holography conFigureurations,

which means that the baseband Fresnel-domain image $\Psi(\xi, \eta)$ can be recovered from the hologram $h(\xi, \eta)$ by using demodulation.



Figure 1. Synthesis of a simulated digital hologram

Figure 1 shows a block diagram of the synthesis proces of a simulated digital hologram. First, an object image $\Phi(x,y)$, which is a two dimensional (complex-valued) function, is defined. The Fresnel transform change the image into Fresnel-domain function $\Psi(\xi,\eta)$, which is then multiplied by a reference wave $r(\xi,\eta)$. At the last stage, additive Gaussian noise $\varepsilon(\xi,\eta)$ simulating thermal fluctuations on the device is added, so that we obtain a simulated noisy digital hologram $h(\xi,\eta)$.

In the reconstruction stage shown in Figure 2, the hologram is first demodulated by multiplying it with the reference wave $r(\xi, \eta)$ and followed by a lowpass filtering. We arbitrarily choose a two-dimensional cosine filter given by

$$f_{c}(\xi,\eta) = \begin{cases} \cos\left(\sqrt{\xi^{2} + \eta^{2}}\right) - 0.5\pi < \xi < 0.5\pi; \\ -0.5\pi < \eta < 0.5\pi \\ 0, \text{ otherwise} \end{cases}$$
(15)

as the lowpass filter. Demodulation process yields a Fresnel-domain image $\hat{\Psi}(\xi,\eta)$ which is an input for the reconstruction algorithm.



Figure 2. Object image reconstruction from a hologram

We should note that we only get an approximate Fresnel-domain image $\hat{\Psi}(\xi,\eta)$, instead of the original one $\Psi(\xi,\eta)$, because the filtering in the demodulation process eliminates some parts of the frequency components. We employed both of the conventional direct inversion based on (10) and then the proposed MPPD algorithm. Finally, the approximation of the object image $\hat{\Phi}(x, y)$ for each algorithm is obtained.

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Figure 3. The Fresnel-domain image: (a) original magnitude (b) original phase, (c) extracted magnitude, (d) extracted phase. Figures (c) and (d) are obtained after demodulation. Filtering makes the later ones different from their originals displayed in (a) and (b).

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Figure 3 shows simulation results of the Fresnel-domain image, before and after filtering in the demodulation process; Figureure (a) and (b) are the magnitude and phase of the original $\Psi(\xi,\eta)$, whereas (c) and (d) are their corresponding approximation of Fresnel-doman image $\hat{\Psi}(\xi,\eta)$. The estimated image shown in (c) and (d) contain less noise and smoother than those ones in (a) and (b).



B. Reconstruction of A Simulated Noisy Hologram

We simulate holographic imaging of four Gaussian particles with various width (variance values) and amplitudes. The hologram is normalized and then contaminated by gaussian noise, both in the real- and imaginary- parts, with the variance (energy) equal to 0.25. We compare reconstruction results of the direct inversion based on Equation (10) with the proposed method. Figure 4 shows (a) the image of the object. Lower right part shows a particle with negative amplitude, while the other three are positive. Since the objects in a holographic imaging generally are complex-valued quantities that possess both magnitude and phase, the negative amplitude represents a complex-valued image whose phase is π , while the positive ones correspond to an image whose phase is zero. The reference wave have non zero component in ξ and η directions. The simulated hologram is displayed in Figure.4 (b).

Figure 5. (a) Error curve of the MP method and (b) sorted and normalized coefficients

Following reconstruction stages displayed in Figure.2, we first extract a Fresnel-domain object image $\hat{\Psi}(\xi,\eta)$ from the hologram. The spectrum of the hologram exhibits some distinct clusters, located mainly in the center and two other ones in conjugate pairs. The transform-domain object wave $\hat{\Psi}(\xi,\eta)$ is extracted from this spectrum by demodulation, i.e., after multiplying the hologram with the reference wave, the spectrum is filtered by a two-dimensional cosine filter.

Then, the extracted $\hat{\Psi}(\xi,\eta)$ from the hologram is used to reconstruct the image. Figure 5 (a) shows the MPPD iteration process in term of the error energy reduction, whereas Figure. 5(b) displays a curve of sorted magnitude-coefficients obtained by the MPPD algorithm. The later Figureure indicates that there are only four dominant coefficients which is in agreement with the number of the simulated particles.

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Figure 6. Reconstruction results: (a) direct inversion from hologram, (b) direct inversion from recovered Fresnel-domain image, (c) proposed method

In low noise regions, where the signal (image) energy is much greater than the noise, both algorithms perform well. However, as the noise energy start to exceed the signal's, direct inversion method degrades quickly, whereas the MPPD's SNR only reduces slightly. The results show that the MPPD method is well-suited for high-noise hologram than the direct inversion method.

No	Noise Variance	SNR (dB)	
		Direct Inversion	MPPD
1	0.00	25.59	25.44
2	0.25	25.10	25.50
3	0.50	23.62	25.34
4	1.00	19.47	25.23
5	2.00	11.85	23.16
6	3.00	7.01	21.69
7	4.00	3.87	21.37
8	5.00	1.40	17.29

Table 1. Robustness to Noise Performance

Figure 6 shows reconstruction results by three different methods: (a) direct inversion by applying (10) directly to the hologram, (b) inverse Fresnel transform of extracted $\hat{\Psi}$, and (c) reconstruction by the proposed method. It is shown in this Figure that inverse Fresnel transform perform better than direct inversion, while the proposed MPPD method outperforms the two other ones.

We also evaluate the performance of the proposed MPPD reconstruction algorithms under various noise conditions. Table 1 displays performance comparison of the algorithms in term of robustness to noise. The reconstruction qualities are expressed in SNR (Signal to Noise Ratio). We generate Gaussian noise with various levels of energy and added it into the real- and imaginary- parts of Fresnel-transformed image $\hat{\Psi}(\xi, \eta)$. Accordingly, the noise affects both of the magnitude and phase parts.

The second column of the Table 1 indicates the values of noise energy (variance). Although the variance is zero, the reconstructed image in the first row has a finite SNR, because the demodulation give us $\hat{\Psi}(\xi,\eta)$ instead of $\Psi(\xi,\eta)$.

C. Reconstruction of an Actual Hologram

In this section, we discuss the application of the proposed method to an actually measured hologram. We use an off-axis hologaphic imaging in transmission mode shown in Figure. 7. Similar imaging system has been proposed by other authors, including the resolution capability of the off-axis holographic technique [20]. This design is implemented in an optical bench, where we use a He-Ne laser (λ =632.8 *nm*) source and a 100× magnifications microscope objective (MO) lens as the main part of the system. The objective lens with 40× magnification is used as a BE (Beam Expander), while the attenuator employing a polarizer is used to adjust the contrast in the hologram. In the Figureure, BS indicates the beam splitter, whereas *M* is the

mirror. The image sensor is a WAT-231S CCD (Charged-Coupled Device) camera working in NTSC mode, so that the unit cell size is $6.35 \mu m(H) \times 7.40 \mu m(V)$, and delivers an image with 480×640 pixel size. We prepared microscopic spherical particle of *silica gel* as an object, whose diameters are distributed within 5-10 μm range.

Figure 7. Off-Axis digital holographic experimental setup in optical bench used in the proposed method

Figure 8 shows the recorded digital hologram in (a) and recovered object image after applying the direct inverse method in (b) and (c). We observed the presence of four objects in the image; each of it has been highly distorted. Then, the MPPD algorithm is applied to the Fresnel domain object image. The program execution stopped after a few number of iterations and the error curve is shown in Figure. 9(a), yields only a few dominant coefficient as shown in Figure 9(b).

The reconstruction result of the hologram is shown in Figure 9(c). In contrast to the direct inversion, the proposed method exhibits fewer distortions, except smearing effect around some of the particles. One particle located in the lower-left part seems to have a correct focus, while the other three are unfocused and exhibit smearing effects. The non-focus case may due to the variation of distance between the screens to each of the object. Therefore, we have to consider a case where the particles lying at different sections, and we have to revise our bases functions to incorporate such condition.

D. Reconstruction Based on 3D Bases and Slice Thresholding

In a 3D bases, the position of the object in a particular layer or section is taken into account, although the transform domain bases will be still lay in a two-dimensional space. This is consistent with the fact that a hologram is a two-dimensional image representing a 3D object. Figureure 10 shows simulation results for a 3D reconstruction; the left parts shows original images of the particles in four different sections, where as the right parts displays reconstruction results. This Figure shows that, if we use a 3D bases, the MPPD will be able to distinguish the object in three-dimensional tomographic sections.

Figure. 8. (a) A hologram and its reconstructed image by direct inversion: (b) magnitude and (c) phase

Figure 9. (a) Error curve, (b) sorted coefficients, and (c) reconstructed image by the MPPD method

Figure.10. Object and its location in four different sections: left part original, right part reconstructed, (a) and (b) layer-1, (c) and (d) layer-2, (e) and (f) leyer-3, (g) and (h) layer-4

Next, we apply the MPPD with 3D bases to a real hologram, where the image of each sections are displayed in Figure.11. Among all of five sections, there are two dominant ones located in layer 3 and layer 4. We compare the result with the direct inversion method obtained previously. First we combine the sections into a single Figure, to compare with the conventional method that assumes a single section.

Figure.11. Object recovered in five sections: Figures. (a) - (e) corresponds to section-1 - section-5.

Based on Figure 12, we observe that the proposed method locates the particles exactly to the center of positions that corresponds to the direct inversion results (ref. Figure.8(b)), but, with much fewer artifacts. Further application of a threshold or selection of sections with high energy can give a sharper image, as shown in Figure. 12 (b). These results show the applicability of the MPPD algorithm to reconstruct 3D tomographic sections based only on a single hologram

Figure.12. Comparison of reconstruction result: (a) overlaid 3D reconstruction using the MPPD method, (b) Further processing by thresholding each layer will sharpen the object image.

4. Conclusion

We have presented a new method to reconstruct object image from a digital hologram. The method uses a pair of domains, i.e. the spatially localized bases and its corresponding Fresnel bases. The Fresnel bases are used to estimate the coefficient values, and then we use the

coefficients to reconstruct the object image in the spatial domain. Further processing with 3D bases and thresholding improves the image results.

For object images with any arbitrary shapes, the bases in the proposed method should be extended into ones consisting of various localized spatial bases. The simulation and real-hologram processing results shows the capability of the MPPD algorithm. In addition, using a better camera with higher resolution and improvement of the imaging system could significantly improve the results.

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