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Robust Decentralized Adaptive Fuzzy Integral Sliding Mode Control of Mismatched Uncertain Large-scale Systems

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Abstract: The objective of this paper is to propose a robust decentralized adaptive fuzzy integral sliding mode control of interconnected uncertain systems. Uncertainties, disturbances and interconnections are of mismatched type. The approach, which will be considered, benefits from the advantages of the integral sliding mode compared to traditional sliding mode. Moreover, it combines the LMI technique with the sliding mode to design a sliding surface guaranteeing the satisfaction of H infinity robustness criterion. The proposed local controllers designed for each subsystem guaranty the quadratic stability of the global system. The second task considered in this work is the synthesis of an adaptive fuzzy control scheme allowing the elimination of the phenomenon of chattering and the estimation of the controller parameters. The effectiveness and the useful of the obtained results will be discussed through numerical example.

Keywords: Decentralized control, integral sliding mode, large-scale systems, LMI, mismatched uncertainties, robustness, adaptive fuzzy control.

1. Introduction

The uncertain modeling of the dynamic systems is much closer to the reality of the practical systems. Indeed, the presence of uncertainties can be due to the variation of the parameters of the system or the inaccuracy of the knowledge of their values. The presence of the nonlinearities, as well as the external disturbances could be also at the origin of this uncertain modeling. So, the control of these systems has attracted the attention of researchers in recent times and has been widely considered. Variable Structure Control (VSC) with Sliding Mode(SM) has a variety of attractive features such a fast response, good transient performance, and order-reduction [1,2]. An additional salient advantage of this control approach is its completely robustness to systems with uncertainties verifying the so-called matching condition [3]. This propriety is at the origin of the vast use of SM in the design of robust control for such class of systems. However, the Classic Sliding Mode Control (CSMC) provides the desired motion after a sliding mode occurs. This insufficiency has been the principal motivation of researchers to proceed to the elimination of the reaching phase. A recent method, based on a particular choice of the switching function, provides the so-called Integral Sliding Mode Control (ISMC) which allows the existence of sliding mode from the initial time [4,5]. Therefore, the system is immediately robust and insensitive to uncertainties and disturbances. The application of ISMC in the control of matched uncertain MIMO systems has resulted in good performances as robustness and tracking response [6]. Inopportunely, both CSMC and ISMC may be unsuccessful in the stabilization of mismatched uncertain systems. Because the system dynamics in sliding mode, opposing to matched uncertain system, are not uncertainties free. To surmount this problem, the main idea is the combination of SMC with other robust techniques. The common existing methods are based on CSMC, so that they are affected by the over mentioned insufficiency of SMC with reaching phase [7–9]. Recently, more attention is focused on the advantages of ISMC in the control of systems with mismatched uncertainties. This method is extended to cover systems with mismatched uncertainties in the state matrix, but it is restricted to matched uncertainties in the input matrix and the external disturbances [10]. Only a few of the more recently studies have included the case of mismatched uncertainties in the input matrix [11,12]. Though, no one of

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them is directly applicable in the presence of the mismatched disturbances and the norm-bounded nonlinearities which are not related to the input channel. An original approach based on ISMC associated to LMI approach and H characterization has been, recently, developed [13,14]. The considered class of uncertain systems can cover large sets of systems; indeed, the mismatched uncertainties in both state and input matrices, the norm-bounded nonlinearities and external disturbances have been investigated by this approach. This method has several advantages such the increase of feasibility domain of the LMI solution [13] and the optimization of the nonlinear control gain enabled by the proposed methodology of uncertainties. Furthermore, the chattering problem has been surmounted by the design of an Adaptive Fuzzy Integral Sliding Mode Controller (AFISMC) [14]. It is generally considered that large-scale and complex systems are very difficult to stabilize with a single controller. This is due to computational complexity caused by large dimensions and effects of interconnections. Therefore, for designing a large-scale control system, the researchers in this field often divided the entire system into several subsystems, and utilized the decentralized controller to stabilize each subsystem [15]. Many works based on SMC have been carried out in the goal to establish decentralized control schemes. When the system contains only matched uncertainties and interconnections, the known salient advantages of CSMC have been also verified for this class of large-scale systems [16,17]. Moreover, the decentralized SMC of interconnected nonlinear has been considered. Indeed, the case of nonlinear systems in regular form has been envisaged in [18,19] and the control of complex large-scale systems with non-smooth nonlinearities has been carried out in [20]. The advantages and the faculties of SMC have been confirmed in [21] by the real-time implementation of decentralized control scheme on a twin-rotor system. The design of decentralized controllers based on ISMC has been also considered and the related benefits have been preserved such us initial time robustness and invariance in presence of both uncertainties and interconnections [22,23]. When the large-scale system contains mismatched perturbation, a decentralized sliding mode control scheme based on overlapping method is presented in [24]. Only some works consider the case of uncertainties in the input channel [25–28]. Recently, in [29] a decentralized adaptive sliding mode control for large-scale systems with mismatched perturbations has been presented; however, the class of system considered remains restricted. Therefore, the main contribution of the present work is the design of a robust decentralized integral sliding mode control based on LMI technique to guaranty a H_∞ criterion for a more general class of mismatched uncertain large-scale systems. The second task will be the proposition of a decentralized adaptive fuzzy ISMC controller. The goal of the proposed control scheme is the elimination of chattering problem and the relaxation of the necessity knowledge of the exact values of uncertainties and interconnections bounds. In addition, the reachability of sliding surface will be maintained. The proposed paper will be organized as follows. In Section II some useful preliminary results will be given, as well as the system description. Section III will be reserved to the detailed presentation of the proposed approach concerning decentralized integral sliding mode control of mismatched large-scale systems. Mathematical proofs of proposed theoretical results will be given as well as convenient remarks. In section IV, the improvement of the proposed control scheme performances, such as chattering elimination and estimation of norm bounds of uncertainties, is then considered with the application of an adaptive fuzzy integral sliding mode control law. The efficiency of the proposed control laws will be investigated in section V through numerical example. The last section will be allowed to conclusion remarks of the work.

2. Preliminary results and system description

A. Preliminary results

In this section, we give some preliminary results that will be helpful to obtain main results.

Lemma1. [25] Consider the following unforced system:

$$\begin{cases} \dot{x} = A.x + H.w(x,t) \\ y = C.x \end{cases} \tag{1}$$

This system is quadratically stable and satisfies $H_\infty : \|T_{yw}\|_\infty < \gamma$ if there exists a quadratic Lyapunov function $V(x) = x^T P x$, $P > 0$ such that, for all $t > 0$:

$$\dot{V} + y^T y - \gamma^2 w^T w < 0 \tag{2}$$

Lemma2. [11] For any vectors x and y with appropriate dimensions, the following inequality holds:

$$2x^T y \le \alpha x^T x + \alpha^{-1} y^T y, \ \forall \alpha > 0 \tag{3}$$

Lemma3. (Schur complement) [25]: consider a bloc symmetric matrix

$$\begin{bmatrix} A & B^T \\ B & C \end{bmatrix} \tag{4}$$

where A and C are square matrices, with C is negative definite. This matrix is negative definite if and only if $\left(A - B^T C^{-1} B\right)$ is negative semi-definite.

B. System description

We will consider a class of mismatched uncertain large-scale systems composed by N interconnected subsystems E_i:

$$\begin{cases} \dot{x}_i = [A_i + \Delta A_i]x_i + [B_i + \Delta B_i]u_i + f_i(x_i,t) + H_i w_i(t) + \sum_{j \ne i}^{N} h_{ij}(x_j,t) \\ y_i = C_i x_i \end{cases} \tag{5}$$

Where: $x_i \in \mathbb{R}^{n_i}$ is the state, $u_i \in \mathbb{R}^{m_i}$ is the input control, $y_i \in \mathbb{R}^{q_i}$ is the controlled output $f_i(x_i,t) \in \mathbb{R}^{n_i}$ is the vector of nonlinearities and un-modeled dynamics, $w_i(t) \in \mathbb{R}^{p_i}$ is the square-integrable external disturbance. $A_i \in \mathbb{R}^{n_i \times n_i}$ is the scale system characteristic matrix, $B_i \in \mathbb{R}^{n_i \times m_i}$ is the input matrix with full rank m_i, $H_i \in \mathbb{R}^{n_i \times p_i}$ is the matrix of external disturbance, $C_i \in \mathbb{R}^{q_i \times n_i}$ is the output matrix. $\Delta A_i(t)$ and $\Delta B_i(t)$ represent the system matrix uncertainty and the input matrix uncertainty, respectively. $h_{ij}(x_j,t) \in \mathbb{R}^{n_i}$ represents the interconnection term specifying the action of the subsystem E_j on the subsystem E_i. We will assume the following to be valid.

A1) The pair $\left(A_i, B_i\right)$is stabilizable.

A2) There exist known constants α_i ,β_i , γ_i , w_{i0} and α_{ij} such that : $\|\Delta A_i\| \le \alpha_i$, $\|\Delta B_i\| \le \beta_i$,$\|f_i(x_i,t)\| \le \gamma_i \|x_i\|$, $\|w_i(t)\| \le w_{i0}$ and $\|h_i(x_i,t)\| \le \alpha_{ij} \|x_j\|$.

A3) $\|B_i^+ \Delta B_i\| \le \beta_{i,m} < 1$ where $\beta_{i,m}$ is a positive known scalar and $B_i^+ \triangleq \left(B_i^T B_i\right)^{-1} B_i^T$.

3. Decentralized integral sliding mode control

A. Switching surface choice

Let us choose the switching function as follows:

$$S_i(t) = B_i^+ x_i + z_i \tag{6}$$

where $z_i \in \mathbb{R}^{m_i}$, is the solution of the following dynamic equation:

$$\dot{z}_i = -B_i^+ \left[A_i + B_i K_i\right] x_i,\; z_i(0) = -B_i^+ x_i(0) \tag{7}$$

where: $K_i \in \mathbb{R}^{m_i \times n_i}$, is a state feedback gain which should be designed, later, to lead to the closed loop system the desired performances in sliding mode.
The considered sliding surface allows the elimination of the reaching phase characterizing the classic sliding mode control, because the initial value $S_i(0) = 0$, for any initial conditions.

When the state trajectories of the system enter the sliding mode, we have $S_i(t) = 0$, and $\dot{S}_i(t) = 0$. The time derivative of the switching function is derived as follows:

$$\dot{S}_i = B_i^+ \left[\Delta A_i x_i + \left(B_i + \Delta B_i\right)u_i + f_i(x,t)\right] + B_i^+ H_i w_i + B_i^+ \sum_{j \neq i}^{N} h_{ij}(x_j,t) - K_i x_i \tag{8}$$

Let us suppose that:

$$\Gamma_i = I_{n_i} - B_i B_i^+ \tag{9}$$

where $I_{n_i} \in \mathbb{R}^{n_i \times n_i}$ is the identity matrix. Accordingly, it is easy to deduce that:

$$B_i^+ \Gamma_i = B_i^+ - B_i^+ B_i B_i^+ = B_i^+ - B_i^+ = 0 \tag{10}$$

In addition, we can rewrite the uncertainty terms as follows:

$$\begin{cases} \Delta A_i(t) = B_i \Delta A_{i,m}(t) + \Delta A_{i,u}(t), \\ \Delta B_i(t) = B_i \Delta B_{i,m}(t) + \Delta B_{i,u}(t), \\ f_i(x_i,t) = B_i f_{i,m}(x_i,t) + f_{i,u}(x_i,t) \\ H_i = B_i H_{i,m} + H_{i,u}, \\ h_{ij}(x_j,t) = B_i h_{ij,m}(x_j,t) + h_{ij,u}(x_j,t). \end{cases} \tag{11}$$

where:

$$\begin{cases} \Delta A_{i,m} = B_i^+ \Delta A_i, \Delta A_{i,u} = \Gamma_i \Delta A_i, \\ \Delta B_{i,m} = B_i^+ \Delta B_i, \Delta B_{i,u} = \Gamma_i \Delta B_i, \\ f_{i,m} = B_i^+ f_i, f_{i,u} = \Gamma_i f_i, \\ H_{i,m} = B_i^+ H_i, H_{i,u} = \Gamma_i H_i, \\ h_{ij,m} = B_i^+ h_{ij}, h_{ij,u} = \Gamma_i h_{ij}. \end{cases} \tag{12}$$

Furthermore, there exist known positive constants $\alpha_{i,m}$, $\alpha_{i,u}$, $\beta_{i,u}$, $\gamma_{i,m}$, $\gamma_{i,u}$, $\alpha_{ij,m}$ and $\alpha_{ij,u}$ such that:

$$\begin{cases} \left\|\Delta A_{i,m}\right\| \le \alpha_{i,m}, \left\|\Delta A_{i,u}\right\| \le \alpha_{i,u}, \\ \left\|\Delta B_{i,m}\right\| \le \beta_{i,m}, \left\|\Delta B_{u}\right\| \le \beta_{i,u}, \\ \left\|f_{i,m}\right\| \le \gamma_{i,m}\left\|x_i\right\|, \left\|f_{i,u}\right\| \le \gamma_{i,u}\left\|x_i\right\|, \\ \left\|h_{ij,m}\right\| \le \alpha_{ij,m}\left\|x_j\right\|, \left\|h_{ij,u}\right\| \le \alpha_{ij,u}\left\|x_j\right\|. \end{cases} \tag{13}$$

Consequently, we can rewrite the derivative of the sliding surface as follows:

$$\dot{S}_i = \Delta A_{i,m} x_i + \left(I_{m_i} + \Delta B_{i,m}\right) u_i + f_{i,m}(x_i) + H_{i,m} w_i + \sum_{j \neq i}^{N} h_{ij,m}(x_j) - K_i x_i \tag{14}$$

Then, we can derive the following expression of the equivalent control:

$$u_{i,eq} = -\left(I_{m_i} + \Delta B_{i,m}\right)^{-1} \left[\Delta A_{i,m} x_i + f_{i,m} + H_{i,m} w_i + \sum_{j \neq i}^{N} h_{ij,m} - K_i x_i\right] \tag{15}$$

Remark 1. Equation (15) requires that the matrix $\left(I_{m_i} + \Delta B_{i,m}\right)$ be nonsingular. This requirement is guaranteed by assumption A3.

B. Sliding mode stability

he sliding mode dynamics are obtained by substituting (15) in (5):

$$\dot{x}_i = A_i x_i + B_i K_i x_i + \tilde{B}_i K_i x_i + \Delta A_{i,u} x_i + f_{i,u} - \tilde{B}_i \left[\Delta A_{i,m} x_i + f_{i,m} + H_{i,m} w_i + \sum_{j \neq i}^{N} h_{ij,m}\right] + H_{i,u} w_i + \sum_{j \neq i}^{N} h_{ij,u} \tag{16}$$

where:

$$\tilde{B}_i = \Delta B_{i,u} \left(I_{m_i} + \Delta B_{i,m}\right)^{-1} \tag{17}$$

The system dynamics in the sliding mode are affected by the existence of uncertainties and disturbances. Thus, although the sliding mode acts correctly in the sense of eliminating the effect of the matched uncertainties and disturbances; the mismatched parts of uncertainties not eliminated by the sliding mode alone requires the use of another robust control technique to mitigate their effect on closed loop system. For this reason, the H_∞ approach is used alongside the sliding mode to accomplish the control requirement. Therefore, the objective of this section is the design of a state feedback gain K_i for every subsystem E_i. This gain guarantees the stability of the closed loop system while satisfying the H_∞ constraint $\|y_i\|_\infty \le \gamma_i \|w_i\|_\infty$. To reach this goal, we proceed by means of the LMI method.

Theorem 1. Consider the uncertain large-scale system (5) with assumption (A1) -(A3), and the over mentioned switching surface (6). For every subsystem E_i, if there exists symmetric matrix $X_i > 0$, matrix R_i and positives scalars $\varepsilon_{k,i}$, $k = 1,\dots,6$, ε_N and γ_i, satisfying:

$$\begin{bmatrix} \Xi_i & \Theta_i \\ * & \Psi_i \end{bmatrix} < 0 \tag{18}$$

where:

$$\Theta_i = \begin{bmatrix} \beta_{i,u} R_i^T & \mu_{1,i} X_i & \mu_{2,i} X_i & H_{i,u} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{19}$$

$$\Psi_i = \begin{bmatrix} -\varepsilon_{5,i}^* I_i & 0 & 0 & 0 & 0 \\ * & -\varepsilon_N I_i & 0 & 0 & 0 \\ * & * & -\varepsilon_N I_i & 0 & 0 \\ * & * & * & -\gamma_i^2 & \beta_{i,u} H_{i,m}^T \\ * & * & * & * & -\varepsilon_{6,i}^* I_i \end{bmatrix} \tag{20}$$

$$\Xi_i = \begin{bmatrix} \Sigma_i & X_i C_i^T & \alpha_{i,u} X_i & \gamma_{i,u} X_i & \beta_{i,u}\alpha_m X_i & \beta_{i,u}\gamma_{i,m} X_i \\ * & -I_i & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_{1,i} I_i & 0 & 0 & 0 \\ * & * & * & -\varepsilon^*_{2,i} I_i & 0 & 0 \\ * & * & * & * & -\varepsilon_{3,i} I_i & 0 \\ * & * & * & * & * & -\varepsilon^*_{4,i} I_i \end{bmatrix} \tag{21}$$

$$\Sigma_i = A_i X_i + X_i A_i^T + B_i R_i + R_i^T B_i^T + \left(\sum_{k=1}^{6} \varepsilon_{k,i} + \varepsilon_N \left(1 + \frac{\beta_{i,u}^2}{\left(1-\beta_{i,m}\right)^2} \right) \right) I_i \tag{22}$$

$$\varepsilon^*_{k,i} = \left(1-\beta_{i,m}\right)^2 \varepsilon_{k,i} , \tag{23}$$

$$m_{1,i} = \sqrt{\left(N-1\right) \sum_{j \neq i}^{N} \alpha_{ji,m}} , \tag{24}$$

$$m_{2,i} = \sqrt{\left(N-1\right) \sum_{j \neq i}^{N} \alpha_{ji,u}} , \tag{25}$$

$$R_i = K_i X_i \tag{26}$$

then, the sliding mode plane is quadratically stable and the given H_∞ performance is valid.

Proof. Consider positive-definite matrices $P_i, i = 1,..,N$ and choose a candidate Lyapunov function,

$$V(x) = \sum_{i=1}^{N} x_i^T P_i x_i \tag{27}$$

To complete the proof, we proceed by verification of lemma 1:

$$\begin{aligned} \dot V + \sum_{i=1}^{N} \left(y_i^T y_i - \gamma_i^2 w_i^T w_i \right) &= \sum_{i=1}^{N} \Big\{ x_i^T \left[P_i A_i + A_i^T P_i + P_i B_i K_i + K_i^T B_i^T P_i^T \right] x_i + 2 x_i^T P_i \Delta A_{u,i} x_i - 2 x_i^T P_i \tilde B_i \Delta A_{m,i} x_i + 2 x_i^T P_i f_{u,i} \\ &\quad - 2 x_i^T P_i \tilde B_i f_{m,i} + 2 x_i^T P_i B_i K_i x_i + 2 x_i^T P_i H_{u,i} w_i - 2 x_i^T P_i \tilde B_i H_{m,i} w_i + 2 x_i^T P_i \left(\sum_{j \neq i}^{N} \left(h_{u,ij} - \tilde B_i h_{m,ij} \right) \right) \\ &\quad + x_i^T C_i^T C_i x_i - \gamma_i^2 w_i^T w_i \Big\} \end{aligned}$$

Using lemma 2, we get

$$2 x_i^T P_i \Delta A_{u,i} x_i \le \varepsilon_{1,i} x_i^T P_i^2 x_i + \varepsilon_{1,i}^{-1} x_i^T \Delta A_{u,i}^T \Delta A_{u,i} x_i \le x_i^T \left[\varepsilon_{1.i} P_i^2 + \alpha_{u.i}^2 \varepsilon_{1.i}^{-1} I_i \right] x_i$$

$$-2 x_i^T P_i \tilde B_i \Delta A_{m,i} x_i \le \varepsilon_{2,i} x_i^T P_i^2 x_i + \varepsilon_{2,i}^{-1} x_i^T \Delta A_{m,i}^T \tilde B_i^T \tilde B_i \Delta A_{m,i} x_i \le x_i^T \left[\varepsilon_{2.i} P_i^2 + \frac{\beta_{u,i}^2 \alpha_{m.i}^2}{\left(1-\beta_{m,i}\right)^2} \varepsilon_{2.i}^{-1} I_i \right] x_i$$

$$2 x_i^T P_i f_{u,i} \le \varepsilon_{3,i} x_i^T P_i^2 x_i + \varepsilon_{3,i}^{-1} f_{u,i}^T f_{u,i} \le x_i^T \left[\varepsilon_{3.i} P_i^2 + \gamma_{u.i}^2 \varepsilon_{3.i}^{-1} I_i \right] x_i$$

$$-2x_i^T P_i \tilde{B}_i f_{m,i} \le \varepsilon_{4,i} x_i^T P_i^2 x_i + \varepsilon_{4,i}^{-1} f_{m,i}^T \tilde{B}_i^T \tilde{B}_i f_{m,i} \le x_i^T \left[\varepsilon_{4.i} P_i^2 + \frac{\beta_{u,i}^2 \gamma_{m.i}^2}{\left(1-\beta_{m,i}\right)^2} \varepsilon_{4.i}^{-1} I_i \right] x_i$$

$$2x_i^T P_i \tilde{B}_i K_i x_i \le \varepsilon_{5,i} x_i^T P_i^2 x_i + \varepsilon_{5,i}^{-1} x_i^T K_i^T \tilde{B}_i^T \tilde{B}_i K_i x_i \le x_i^T \left[\varepsilon_{5.i} P_i^2 + \frac{\beta_{u,i}^2 \gamma_{m.i}^2}{\left(1-\beta_{m,i}\right)^2} \varepsilon_{5.i}^{-1} K_i^T K_i \right] x_i$$

$$-2x_i^T P_i \tilde{B}_i H_{m,i} w_i \le \varepsilon_{6,i} x_i^T P_i^2 x_i + \varepsilon_{6,i}^{-1} w_i^T H_{m,i}^T \tilde{B}_i^T \tilde{B}_i H_{m,i} w_i \le x_i^T \varepsilon_{6.i} P_i^2 x_i + \frac{\beta_{u,i}^2}{\left(1-\beta_{m,i}\right)^2} \varepsilon_{6.i}^{-1} w_i^T H_{m,i}^T H_{m,i} w_i$$

$$2x_i^T P_i h_{u,ij}(x_j) \le \frac{\varepsilon_N}{N-1} x_i^T P_i^2 x_i + \varepsilon_N^{-1}\left(N-1\right) \alpha_{u,ij}^2 x_j^T x_j$$

$$2x_i^T \sum_{j \ne i}^{N} P_i h_{u,ij}(x_j) \le \varepsilon_N x_i^T P_i^2 x_i + \varepsilon_N^{-1}\left(N-1\right) \sum_{j \ne i}^{N} \alpha_{u,ij}^2 x_j^T x_j$$

$$-2x_i^T P_i \tilde{B}_i h_{m,ij}(x_j) \le \frac{\varepsilon_N}{N-1} \frac{\beta_{u,i}^2}{\left(1-\beta_{m,i}\right)^2} x_i^T P_i^2 x_i + \varepsilon_N^{-1}\left(N-1\right) \alpha_{m,ij}^2 x_j^T x_j$$

Consequently, we obtain

$$\dot{V} + \sum_{i=1}^{N} \left(y_i^T y_i - \gamma_i^2 w_i^T w_i \right)$$
$$\le \sum_{i=1}^{N} \left\{ x_i^T \Omega_i x_i + x_i^T P_i H_{u,i} w_i + w_i^T H_{u,i}^T P_i x_i + w_i^T \left(-\gamma_i^2 I_i + \frac{\beta_{u,i}^2}{\left(1-\beta_{m,i}\right)^2} \varepsilon_{6.i}^{-1} H_{m,i}^T H_{m,i} \right) w_i \right\} \tag{28}$$

Where

$$\Omega_i = P_i A_i + A_i^T P_i + P_i B_i K_i + K_i^T B_i^T P_i + \left(\sum_{k=1}^{6} \varepsilon_{k,i} + \varepsilon_N \left(1 + \frac{\beta_{u,i}^2}{\left(1-\beta_{m,i}\right)^2} \right) \right) P_i^2$$
$$+ \left(\alpha_{u,i}^2 \varepsilon_{1,i}^{-1} + \frac{\beta_{u,i}^2 \alpha_{m,i}^2}{\left(1-\beta_{m,i}\right)^2} \varepsilon_{2,i}^{-1} + \gamma_{u,i}^2 \varepsilon_{3,i}^{-1} + \frac{\beta_{u,i}^2 \gamma_{m,i}^2}{\left(1-\beta_{m,i}\right)^2} \varepsilon_{4,i}^{-1} + \varepsilon_N^{-1}\left(N-1\right) \sum_{j \ne i}^{N} \left[\alpha_{u,ji}^2 + \alpha_{m,ji}^2 \right] \right) I_i$$
$$+ \frac{\beta_{u,i}^2}{\left(1-\beta_{m,i}\right)^2} \varepsilon_{5,i}^{-1} K_i^T K_i \tag{29}$$

then, we can rewrite that

$$\dot{V} + \sum_{i=1}^{N} \left(y_i^T y_i - \gamma_i^2 w_i^T w_i \right) \le \sum_{j \ne i}^{N} \left\{ \begin{bmatrix} x_i^T & w_i^T \end{bmatrix} \Phi_i \begin{bmatrix} x_i \\ w_i \end{bmatrix} \right\} \tag{30}$$

with

$$\Phi_i = \begin{bmatrix} \Omega_i & P_i H_{u,i} \\ H_{u,i}^T P_i & -\gamma_i^2 I_i + \frac{\beta_{u,i}^2}{\left(1-\beta_{m,i}\right)^2} \varepsilon_{6.i}^{-1} H_{m,i}^T H_{m,i} \end{bmatrix} \tag{31}$$

thus, lemma 1 is satisfied if, for every subsystem E_i, we have

$$\Phi_i < 0 \tag{32}$$

The next inequality can be derived from the last one by using lemma 3:

$$\begin{bmatrix} \Omega_i & P_i H_{u,i} & 0 \\ H_{u,i}^T P_i & -\gamma_i^2 I_i & \beta_{u,i} H_{m,i}^T \\ 0 & \beta_{u,i} H_{m.i} & -\left(1-\beta_{m,i}\right)^2 \varepsilon_{6,i} \end{bmatrix} < 0 \tag{33}$$

After, pre-multiplying and post-multiplying (33) by $diag\left\{P_i^{-1}, I_i, I_i\right\}$, considering $X_i = P_i^{-1}$ and $R_i = K_i X_i$, the LMI (18) is obtained by a successive use of lemma 3. Therefore, the Proof is achieved.

C. Decentralized controller design

Now, we proceed to the second task which is the design of the sliding mode control law enabling the reachability to the specified switching function.

Theorem 2. Consider the uncertain large-scale system with (5) assumption (A1) -(A3), and the switching surface (6). Suppose that, for every subsystem E_i, the SMC law is:

$$u_i = K_i x_i - \rho_i \frac{S_i}{\left\| S_i \right\|} \tag{34}$$

where:

$$\rho_i = \frac{1}{\left(1-\beta_{m.i}\right)} \rho_{1,i}\ , \tag{35}$$

$$\rho_{1,i} = \left(\alpha_{i,m} + \beta_{i,m}\left\|K_i\right\| + \gamma_{i,m} + \sum_{j \neq i}^{N} \alpha_{ji,m}\right)\left\|x_i\right\| + \theta_i + \left\|H_{i,m}\right\| \omega_{i0} \tag{36}$$

with θ_i is a small positive scalar. Then, a stable sliding mode exists from initial time.

Proof. Consider the Lyapunov function $V = \sum_{i=1}^{N} \left\| S_i \right\|$ which is positive definite. The derivative of this function respect to time is :

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{N} \frac{S_i^T \dot{S}_i}{\left\|S_i\right\|} = \sum_{i=1}^{N} \frac{S_i^T}{\left\|S_i\right\|}\left[\Delta A_{i,m} x_i + \left(I_{m_i} + \Delta B_{i,m}\right) u_i + f_{i,m} + H_{i,m} w_i + \sum_{j \neq i}^{N} h_{ij,m} - K_i x_i\right] \\
&= \sum_{i=1}^{N} \frac{S_i^T}{\left\|S_i\right\|}\left[\Delta A_{i,m} x_i + f_{i,m} + H_{i,m} w_i + \sum_{j \neq i}^{N} h_{ij,m} + \left(I_{m_i} + \Delta B_{i,m}\right)\left(K_i x_i - \rho_i \frac{S_i}{\left\|S_i\right\|}\right) - K_i x_i\right] \\
&= \sum_{i=1}^{N} \frac{S_i^T}{\left\|S_i\right\|}\left[\Delta A_{i,m} x_i + f_{i,m} + H_{i,m} w_i + \sum_{j \neq i}^{N} h_{ij,m} - \rho_i \frac{S_i}{\left\|S_i\right\|} + \Delta B_{i,m} K_i x_i - \rho_i \Delta B_{i,m} \frac{S_i}{\left\|S_i\right\|}\right] \\
&= \sum_{i=1}^{N} \left\{ \frac{S_i^T}{\left\|S_i\right\|}\left[\Delta A_{i,m} x_i + f_{i,m} + H_{i,m} w_i + \sum_{j \neq i}^{N} h_{ij,m} + \Delta B_{i,m} K_i x_i\right] - \rho_i - \rho_i \Delta B_{i,m} \right\} \\
&\leq \sum_{i=1}^{N} \left\{ \left[\left\|\Delta A_{i,m}\right\|\left\|x_i\right\| + \left\|f_{i,m}\right\| + \left\|H_{i,m}\right\|\left\|w_i\right\| + \sum_{j \neq i}^{N}\left\|h_{ij,m}\right\| + \left\|\Delta B_{i,m}\right\|\left\|K_i\right\|\left\|x_i\right\|\right] - \frac{\rho_{1,i}}{1-\beta_{m,i}} + \frac{\rho_{1,i}}{1-\beta_{m,i}}\left\|\Delta B_{i,\ldots}\right. \right. \\
&\leq \sum_{i=1}^{N} \left\{ \left[\alpha_{i,m}\left\|x_i\right\| + \gamma_{i,m}\left\|x_i\right\| + \left\|H_{i,m}\right\| w_{io} + \sum_{j \neq i}^{N}\left\|h_{ij,m}\right\| + \beta_{i,m}\left\|K_i\right\|\left\|x_i\right\|\right] - \frac{\rho_{1,i}}{1-\beta_{i,m}} + \frac{\rho_{1,i}\beta_{i,m}}{1-\beta_{i,m}} \right\}
\end{aligned}$$

Or:

$$\sum_{i=1}^{N}\sum_{j\neq i}^{N}\left\|\eta_{ij,m}\right\| \leq \sum_{i=1}^{N}\sum_{j\neq i}^{N}\alpha_{ij,m}\left\|x_j\right\| = \sum_{i=1}^{N}\sum_{j\neq i}^{N}\alpha_{ji,m}\left\|x_i\right\|$$

Finally, we get:

$$\dot{V} \leq -\sum_{i=1}^{N}\theta_i < 0$$

Then, the considered SMC law drives the system trajectory into sliding surface in finite time which concludes the proof.

Remark 2. The results proposed in this work represent a generalization of these considered recently for the case of mismatched uncertain systems [13-14]. Indeed, we can find the same results if we consider that N=1. The actual approach inherits same advantages of [13] and [14]. Moreover, this controller gives an efficient decentralized scheme making in consideration the presence of mismatched interconnections.

4. Decentralized adaptive fuzzy integral sliding mode control

To avoid the chattering problem induced by the switching nature of the sliding mode controller, the most common method used in literature is the approximation of the sign function existing in the control expression by a saturation function. However, the linear nature of the last function in the boundary layer affects the closed loop system stability. Consequently, the robustness of the sliding mode control cannot be preserved. So, to eliminate chattering presence with preservation of the robustness characterization of the control, we propose in this section the enhancement of the control procedure by introduction of Fuzzy Logic (FL). After the presentation of the adequate fuzzy mechanism, a Decentralized Adaptive Fuzzy Integral Sliding Mode Control (DAFISMC) will be proposed.

A. Fuzzy mechanism

The proposed DAFISMC is based on the introduction of a Fuzzy Logic inference mechanism which replaces the switching control law. For every subsystem, the switching function can be written as

$$S_i = \begin{bmatrix} s_{i,1} & \cdots & s_{i,k} & \cdots & s_{i,m_i} \end{bmatrix}^T \tag{37}$$

Let $s_{i,k}$ be the input linguistic variable of FL, and $u_{Fi,k}$ be the output linguistic variable. The associated fuzzy sets are expressed as follows:

- for the antecedent proposition ($s_{i,k}$): P (Positive), N (Negative), and Z (Zero);
- for the consequent proposition ($u_{Fi,k}$): PE (Positive Effort), NE (Negative Effort), and ZE (Zero Effort).

To make the sliding surface attractive, the fuzzy linguistic rule base can be given as follows:

1. Rule 1: If $s_{i,k}$ is P, then $u_{Fi,k}$ is PE.
2. Rule 2: If $s_{i,k}$ is Z, then $u_{Fi,k}$ is ZE.
3. Rule 3: If $s_{i,k}$ is N, then $u_{Fi,k}$ is NE.

The membership functions of the input fuzzy sets are of the triangle type, and those of the output fuzzy sets are of the singleton type. The singleton defuzzification method is used in this work. Then the fuzzy controller (output of the defuzzification module) can be written as:

$$u_{Fi,k} = \frac{\sum_{l=1}^{3} \mu_{ik,l}\delta_{ik,l}}{\sum_{l=1}^{3} \mu_{ik,l}} \tag{38}$$

Where:

$0 \le \mu_{ik,l} \le 1\,0$ is the firing strength of rule $l, l = 1,..,3$,

$\delta_{ik,1} = \delta_{ik}, \delta_{ik,2} = 0, \delta_{ik,3} = -\delta_{ik}$ stand for the centers of the membership functions PE, ZE, and NE, respectively.

Owing to the special choice of triangular membership functions, we get $\sum_{l=1}^{3} \mu_{ik,l} = 1$. As a result, we can deduce the following:

$$u_{Fi,k} = \left(\mu_{ik,1} - \mu_{ik,3}\right)\delta_{ik} \tag{39}$$

B. Decentralized adaptive fuzzy controller design

In this section, we interest to the design of a DAISMC which use the output of the precedent inference mechanism. The following theorem describes the adaptive fuzzy control law which guaranties the reachability to sliding surface and the elimination of chattering.

Theorem 3. Consider the uncertain large-scale system with assumption (A1) -(A3), and the over mentioned switching surface. Suppose that, for every subsystem E_i, the DAFISMC law is:

$$u_i = K_i x_i + \hat{u}_{Fi} \tag{40}$$

where:

$$\begin{cases} \hat{u}_{Fi} = \dfrac{-1}{1-b_{i,m}}\begin{bmatrix}\hat{u}_{Fi,1} & \cdots & \hat{u}_{Fi,k} & \cdots & \hat{u}_{Fi,m_i}\end{bmatrix}^T, \\ \hat{u}_{Fi,k} = \left(\mu_{ik,1} - \mu_{ik,3}\right)\hat{\delta}_{ik}. \end{cases} \tag{41}$$

$\hat{\delta}_{ik}$ is given by the following adaptive law:

$$\hat{\delta}_{ik} = b_{ik}\left|\mu_{ik,1} - \mu_{ik,2}\right|, k = 1,..,m_i \tag{42}$$

then, a stable sliding mode exists from initial time.

Proof. First, we consider that controller applied to the system is the fuzzy one given by:

$$u_{Fi} = \frac{-1}{1-b_{i,m}}\begin{bmatrix}u_{Fi,1} & \cdots & u_{Fi,k} & \cdots & u_{Fi,m_i}\end{bmatrix}^T \tag{43}$$

Let us consider the following Lyapunov candidate function:

$$V = \sum_{i=1}^{N} \left\|S_i\right\|_1 = \sum_{i=1}^{N} sign(S_i)^T S_i \tag{44}$$

where: $\left\|.\right\|_1$ denotes the norm 1 of a vector and $sign(S_i)^T = \begin{bmatrix} sign(s_{i,1}) & \cdots & sign(s_{i,m_i})\end{bmatrix}$.

The time derivative of this Lyapunov function is given by

$$\dot{V} = \sum_{i=1}^{N} sign(S_i)^T \dot{S}_i = \sum_{i=1}^{N} sign(S_i)^T \left[\Delta A_{i,m}x_i + \left(I_{m_i} + \Delta B_{i,m}\right)u_i + f_{i,m} + H_{i,m}w_i + \sum_{j \ne i}^{N} h_{ij,m} - K_i x_i\right]$$

$$= \sum_{i=1}^{N} sign(S_i)^T \left(\Delta A_{i,m}x_i + \left(I_{m_i} + \Delta B_{i,m}\right)u_{F,i} + \Delta B_{i,m}K_i x_i + f_{i,m}(x_i,t) + H_{i,m}\omega_i + \sum_{j \ne i}^{N} h_{ij,m}(x_j,t)\right)$$

$$= \sum_{i=1}^{N} sign(S_i)^T \left(\Phi_i \left(x_i ,\ \omega_i \right) + \left(I_{m_i} + \Delta B_{i,m} \right) u_{F,i} + \sum_{j \neq i}^{N} h_{ij,m}(x_j, t) \right)$$

with:

$$\Phi_i \left(x_i ,\ \omega_i \right) = \Delta A_{i,m} x_i + \Delta B_{i,m} K_i x_i + f_{i,m}(x_i, t) + H_{i,m} \omega_i$$

In addition:

$$\sum_{i=1}^{N} sign(S_i)^T \sum_{j \neq i}^{N} h_{ij,m}(x_j, t) = \sum_{i=1}^{N} \sum_{j \neq i}^{N} sign(S_j)^T h_{ji,m}(x_i, t) .$$

Let us define:

$$\psi_i \left(x_i ,\ \omega_i \right) = \Phi_i \left(x_i ,\ \omega_i \right) + \sum_{j \neq i}^{N} sign(S_j)^T h_{ji,m}(x_i, t)$$

then, we can rewrite that:

$$\dot{V} \le \sum_{i=1}^{N} \left\{ \sum_{k=1}^{m_i} \left| \psi_{i,k} \left(x_i ,\ \omega_i \right) \right| + sign(S_i)^T u_{F,i} + sign(S_i)^T \Delta B_{i,m} u_{F,i} \right\}$$

Or, from the definition of fuzzy rules and the expression of the fuzzy controller (37), we can deduce that:

$$sign(S_i)^T u_{F,i} = -\frac{1}{1-b_{i,m}} \sum_{k=1}^{m_i} \left| u_{Fi,k} \right|$$

Consequently, we can deduce that:

$$\dot{V} \le \sum_{i=1}^{N} \left\{ \sum_{k=1}^{m_i} \left| \psi_{i,k} \left(x_i ,\ \omega_i \right) \right| - \frac{1}{1-b_{i,m}} \sum_{k=1}^{m_i} \left| u_{Fi,k} \right| + \frac{b_{i,m}}{1-b_{i,m}} \sum_{k=1}^{m_i} \left| u_{Fi,k} \right| \right\}$$

$$\le \sum_{i=1}^{N} \left\{ \sum_{k=1}^{m_i} \left| \psi_{i,k} \left(x_i ,\ \omega_i \right) \right| - \sum_{k=1}^{m_i} \left| u_{Fi,k} \right| \right\} = \sum_{i=1}^{N} \left\{ \sum_{k=1}^{m_i} \left| \psi_{i,k} \left(x_i ,\ \omega_i \right) \right| - \sum_{k=1}^{m_i} \left| \left(\mu_{ik,1} - \mu_{ik,3} \right) \right| \delta_{ik} \right\}$$

$$= \sum_{i=1}^{N} \left\{ \sum_{k=1}^{m_i} \left[\left| \psi_{i,k} \left(x_i ,\ \omega_i \right) \right| - \left| \left(\mu_{ik,1} - \mu_{ik,3} \right) \right| \delta_{ik} \right] \right\}$$

As a result, $\dot{V} < 0$ if the following inequality holds:

$$\delta_{ik} > \frac{\left| \psi_{i,k} \left(x_i ,\ \omega_i \right) \right|}{\left| \left(\mu_{ik,1} - \mu_{ik,3} \right) \right|} \tag{45}$$

The validity of the last condition is guaranteed by the existence of an optimal value $\bar{\delta}_{ik}$ as proved in Wang's theorem [31]. The complexity of the function $\psi_i \left(x_i ,\ \omega_i \right)$ and the difficulty in the exact knowledge of uncertainties bounds make the determination of the value $\bar{\delta}_{ik}$ very difficult. In order to surmount this problem, $\bar{\delta}_{ik}$ can be estimated; $\hat{\delta}_{ik}$ is its estimated value.

The fuzzy controller is replaced by the adaptive one mentioned in theorem 3, using the adaptive law (36). The Lyapunov candidate function is modified to become:

$$V_1 = V + \sum_{i=1}^{N} \left\{ \sum_{k=1}^{m_i} \frac{1}{2} \beta_{i,k}^{-1} \tilde{\delta}_{i,k}^2 \right\} \tag{46}$$

With: $\tilde{d}_{ik} = \hat{d}_{ik} - \bar{d}_{ik}$ is the estimation error. The time derivative of the new Lyapunov function is given by:

$$\dot{V}_1 \le \dot{V} + \sum_{i=1}^{N}\left\{\sum_{k=1}^{m_i} \beta_{i,k}^{-1}\tilde{\delta}_{i,k}\dot{\hat{\delta}}_{i,k}\right\} \le \sum_{i=1}^{N}\left\{\sum_{k=1}^{m_i}\Big[\left|\psi_{i,k}\left(x_i,\, \omega_i\right)\right| - \left|\left(\mu_{ik,1}-\mu_{ik,3}\right)\right|\hat{\delta}_{ik} + \tilde{\delta}_{i,k}\left|\mu_{ik,1}-\mu_{ik,3}\right|\Big]\right\}$$

$$= \sum_{i=1}^{N}\left\{\sum_{k=1}^{m_i}\Big[\left|\psi_{i,k}\left(x_i,\, \omega_i\right)\right| - \left|\left(\mu_{ik,1}-\mu_{ik,3}\right)\right|\bar{\delta}_{ik}\Big]\right\} < 0$$

This, achieves the proof.

5. Illustrative example

Consider the following large-scale system with the dynamic equations in the form of (5) (N=2):

Subsystem E1: $n_1 = 2,\ m_1 = 1,$

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0.01 \end{bmatrix}\ B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\ \Delta A_1 = \begin{bmatrix} 0.1\sin(t) & 0 \\ 0.3\sin(t) & 0 \end{bmatrix}\ \Delta B_1 = \begin{bmatrix} 0.2\sin(t) \\ 0.4\sin(t) \end{bmatrix}$$

$$f_1(x_1,t) = \begin{bmatrix} 0.2\sin(x_{11})x_{12} \\ 0.4\sin(x_{12})x_{11} \end{bmatrix}\ H_1 = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix}\ w_1 = \sin(t),\ C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$h_{12}(x_2) = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} x_{21}x_{22},$$

Subsystem E2: $n_2 = 2,\ m_2 = 1,$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\ B_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}\ \Delta A_2 = \begin{bmatrix} 0 & 0.2\cos(t) \\ 0.3\sin(t) & 0 \end{bmatrix}\ \Delta B_2 = \begin{bmatrix} 0.3\cos(t) \\ 0.5\cos(t) \end{bmatrix}$$

$$f_2(x_2,t) = \begin{bmatrix} 0.3\cos(x_{21})x_{22} \\ 0.5\sin(x_{22})x_{21} \end{bmatrix}\ H_2 = \begin{bmatrix} 0.3 \\ 0.6 \end{bmatrix}\ w_2 = \sin(0.5t),\ C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$h_{21}(x_1) = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} x_{11}x_{12},$$

The pseudo-inverses of input matrices are given by $B_1^+ = \begin{bmatrix} 0 & 1 \end{bmatrix}$ $B_2^+ = \begin{bmatrix} 0 & 0.5 \end{bmatrix}$. Then, the application of the procedure of uncertainties decomposition given in section II, allows the determination of the following parameters:

Subsystem E1:

$\alpha_{1,m} = 0.3,\ \alpha_{1,u} = 0.1,\qquad \beta_{1,m} = 0.4,\ \beta_{1,u} = 0.2,\qquad \gamma_{1,m} = 0.4,\ \gamma_{1,u} = 0.2,$

$\alpha_{12,m} = 0.4,\ \alpha_{12,u} = 0.2,$

$H_{1,m} = 0.4,\ H_{1,u} = \begin{bmatrix} 0.3 & 0 \end{bmatrix}^T.$

Subsystem E2:

$$a_{2,m} = 0.15,\ a_{2,u} = 0.2, \qquad b_{2,m} = 0.25,\ b_{2,u} = 0.3, \qquad g_{2,m} = 0.25,\ g_{2,u} = 0.3,$$
$$a_{21,m} = 0.2,$$

$$a_{21,u} = 0.2,\ H_{2,m} = 0.3,\ H_{1,u} = \begin{bmatrix} 0.5 & 0 \end{bmatrix}^T .$$

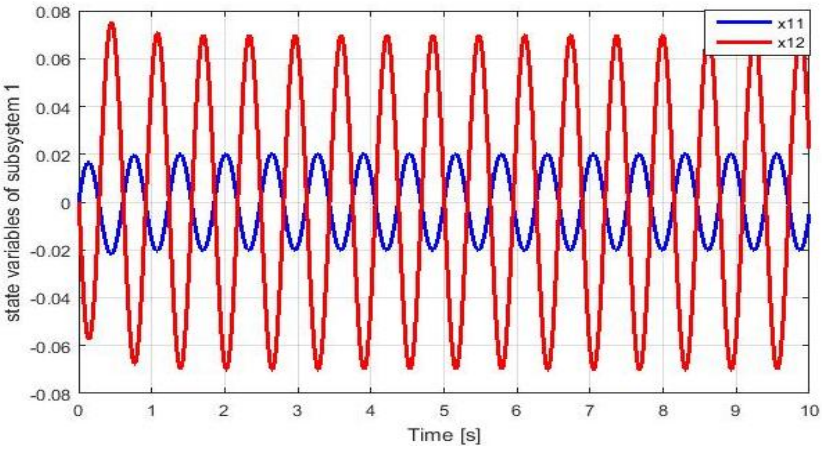
The application of LMI (17) gives the following feasible solutions:
Subsystem E1:

$$X_1 = \begin{bmatrix} 0.2760 & -0.5903 \\ -0.5903 & 1.8359 \end{bmatrix} \quad \mathrm{K}_1 = -\begin{bmatrix} 19.00 & 9.01 \end{bmatrix}\ \gamma_1 = 3,\ \varepsilon_{1,1} = 0.0677,\ \varepsilon_{2,1} = 0.1639,$$
$$\varepsilon_{3,1} = 0.0120,$$

$$\varepsilon_{4,1} = 0.0203,\quad \varepsilon_{5,1} = 0.1054,\quad \varepsilon_{6,1} = 0.0067,\quad \varepsilon_{N,1} = 0.1136$$

Subsystem E2:
$$X_2 = \begin{bmatrix} 0.0987 & -0.2738 \\ -0.2738 & 1.0254 \end{bmatrix} \ \mathrm{K}_2 = -\begin{bmatrix} 15.5 & 5.5 \end{bmatrix}\ \gamma_2 = 2.5,\ \varepsilon_{1,2} = 0.0271,\ \varepsilon_{2,2} = 0.0308,$$
$$\varepsilon_{3,2} = 0.0069,\quad \varepsilon_{4,2} = 0.0093,\quad \varepsilon_{5,2} = 0.0341,\quad \varepsilon_{6,2} = 0.0053,\quad \varepsilon_{N,2} = 0.1136$$

Firstly, we apply the proposed DISMC to the system with null initial conditions, in order to evaluate the H∞ performance. The simulation results are carried out in figure 1 and figure 2. Figure 1 shows the state variables responses which are clearly attenuated; indeed, their magnitudes do not exceed 0.08. This observation is accentuated by actual values of the H∞ performance depicted in figure 2 The related final values are very small.



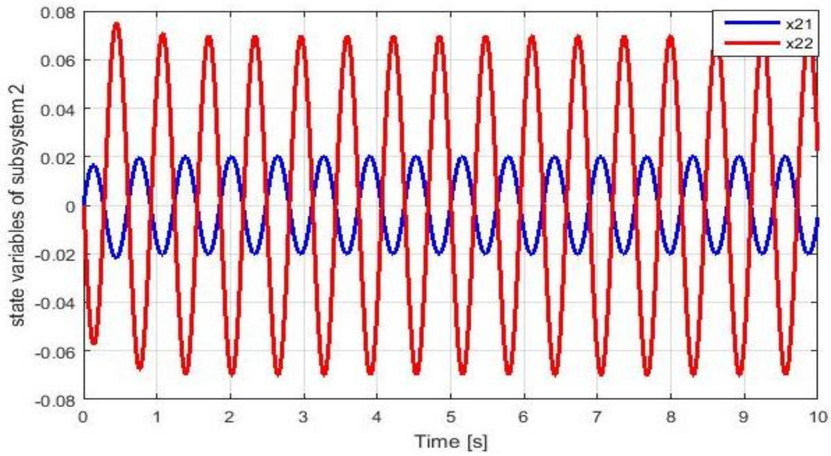


Figure 1. State variables responses with proposed DISMC (null initial conditions)

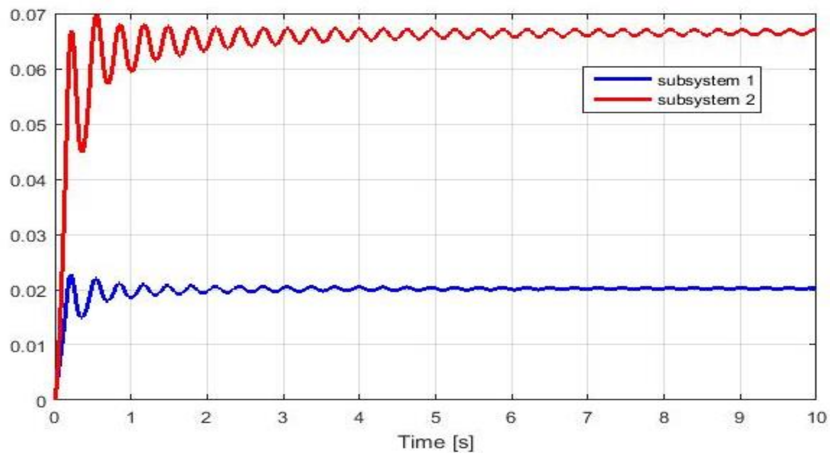


Figure 2. Actual values of H_∞ performance with proposed DISMC

Secondly, we consider the case of no null initial conditions with $x_{10} = \begin{bmatrix} 1 & -1 \end{bmatrix}$ and $x_{20} = \begin{bmatrix} 1 & 1 \end{bmatrix}$ Figure 3 and figure 4 indicate, respectively, the evolution of switching functions and controllers using the proposed DISMC. The same signals, when DAFISMC is applied, are shown in figure 5 and figure 6, respectively. The state variables evolution using DISMC and DAFISMC for both uncertain large-scale system and nominal subsystems (e.g.: in absence of all uncertainties, disturbances and interconnections) are given by figure 7.

From these simulation results, it is obvious that the proposed schemes result in a stable sliding mode from initial time. Though, it is clear from the controller evolution that the DISMC approach is accompanied with chattering phenomenon. This disadvantage is overcome by the DAFISMC over the elimination of high frequency discontinuities in the controller. The state variables responses according to both approaches are superposed. So that, the DAFISMC preserves the same dynamical performances of the closed loop system as the first approach. In addition, these responses are close to small vicinity around those of nominal subsystems, which confirms the robustness of the proposed approaches.

Moreover, to compare the proposed DFISMC approach to the classic method of avoiding chattering phenomenon presumed by the approximation of the sign function by a saturation function, the figures 5, 6 and 7 illustrate also simulation results using this method with a saturation function (with boundary layer of 0.1). It is obvious from the controller evolution in figure 6 that the chattering is avoided. From the evolution of state variables in figure 7, satisfactory results are obtained. However, it is clear from figure 5 that the sliding surfaces are more deviated from the origin which confirm that this method do not preserves the sliding mode stability in the boundary layer because of the linearity of the saturation function around the origin.

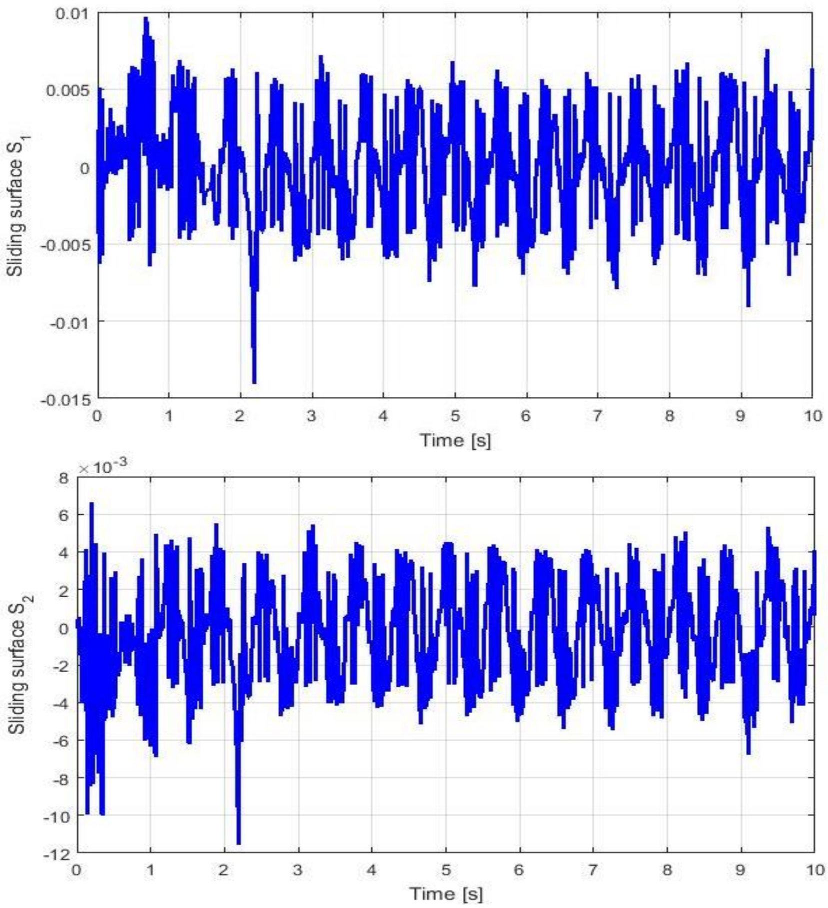
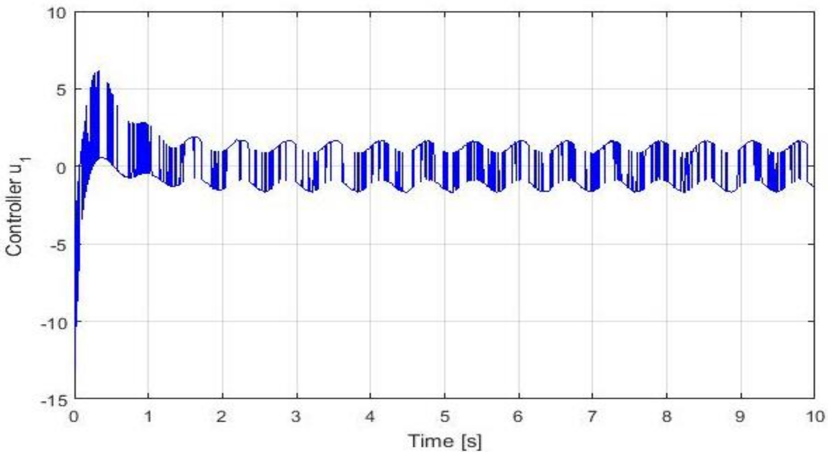


Figure 3. Switching surfaces for both subsystems with proposed DISMC



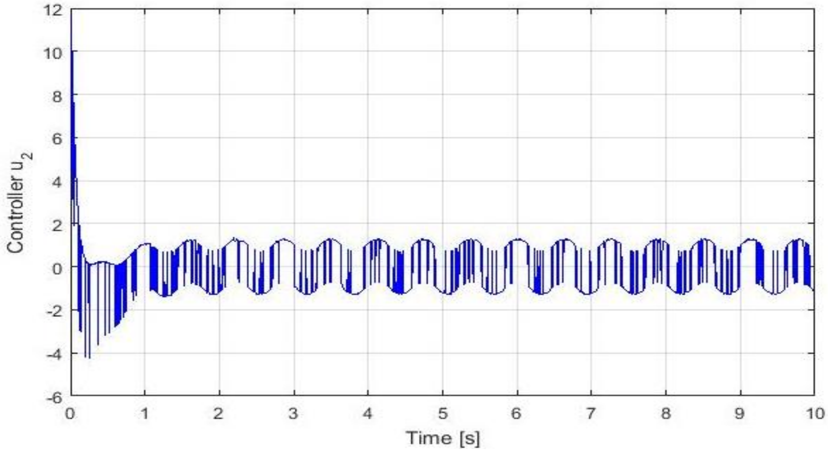


Figure 4. Controllers evolution for both subsystems with proposed DISMC

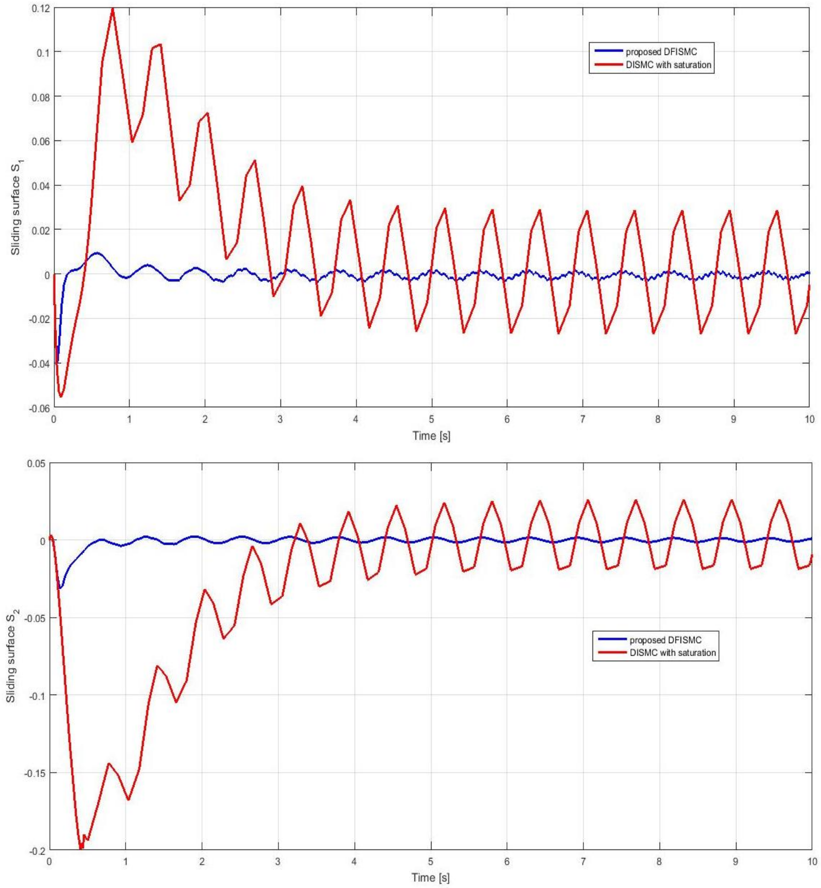


Figure 5. Switching surfaces for both subsystems with proposed DFISMC and DISMC with saturation approximation

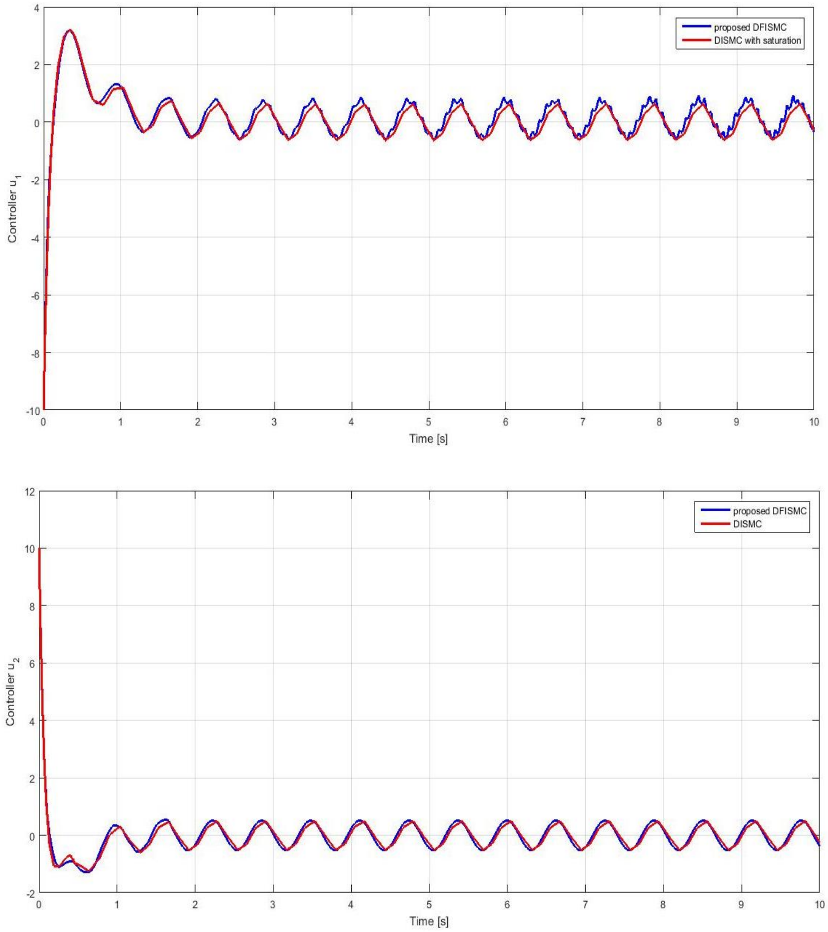
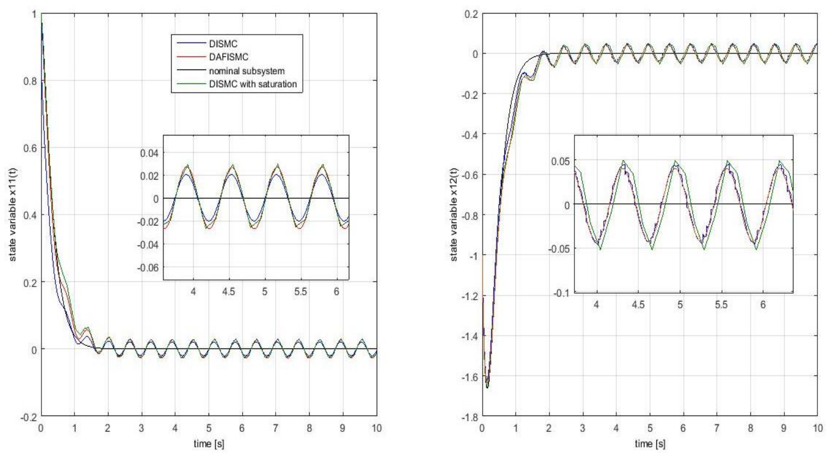


Figure 6. Controllers evolution for both subsystems with proposed DFISMC and DISMC with saturation approximation



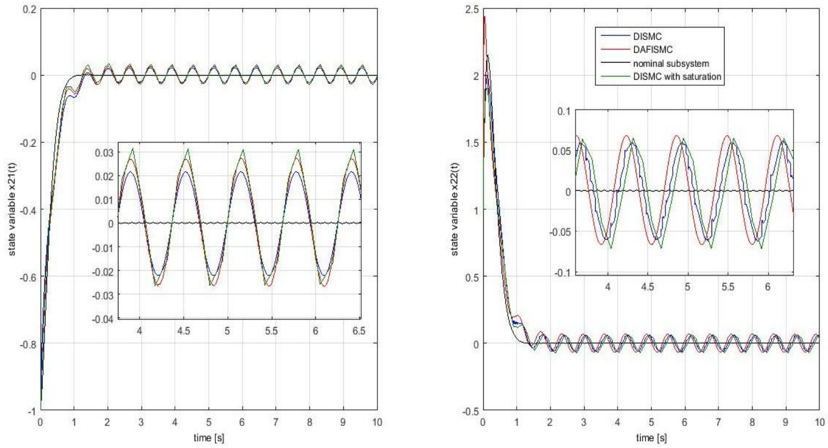


Figure 7. State variables evolution using DISMC, DFISMC and DISMC with saturation approximation for both uncertain large-scale system and nominal subsystems

6. Conclusion

In this paper, the problem of designing robust decentralized integral sliding mode control of large-scale systems with mismatched uncertainties, disturbances and interconnections has been considered. Based on LMI, a sufficient condition of the quadratic stability of sliding mode dynamics with H∞ performance has been proposed. The immediate sliding mode existence has been guaranteed by the proposed control law, and as a result the robustness has been improved. The induced chattering phenomenon has been eliminated by the proposed DAFISMC. The efficacy and the validity of the proposed approaches have been illustrated through numerical examples. Further work will extend the proposed method to large-scale systems with polytopic mismatched uncertainties and nonlinear large-scale systems.

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