



Minimizing Energy of Point Charges on a Sphere using Symbiotic Organisms Search Algorithm

G. Kanimozhi¹, R. Rajathy² and Harish Kumar¹

¹Department of Physics, Pondicherry Engineering College, Pondicherry, India

²Department of EEE, Pondicherry Engineering College, Pondicherry, India
harishkumarholla@pec.edu

Abstract: Determination of the optimal configurations of N identical point charges on a unit sphere, known as Thomson's problem, is a well-known energy optimization problem in physics. In this paper we have determined the minimum energy for various values of N using Symbiotic Organisms Search (SOS) algorithm, up to $N = 200$. The energy values considered in the study are found to be in good agreement with the existing values. Results are presented for various values of N ranging from 3 to 200 along with the convergence graph. We found that SOS algorithm provides better optimal values of energy compared to hybrid approach of Evolutionary algorithm and Spectral Gradient for unconstrained condition (EA_SG2) method.

Keywords: Thomson's problem, Symbiotic Organisms Search, energy minimization, optimal configurations.

1. Introduction

Determination of the optimal configuration of N point charges on a unit sphere is one of the well-known and unsolved problems in electrostatics. This is called Thomson's problem, named after the physicist Joseph John Thomson [1], who, in 1904, studied a related but different arrangement of point charges in his plum pudding atom model to demonstrate the atomic structure. In this model, the repulsive point charges were the electrons embedded into a positive sphere which formed spherical shells, repelling each other with a force given by Coulomb's law. Thus, the potential energy of the system is:

$$U = \frac{q_e^2}{4\pi\epsilon_0} \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{1}{r_{ij}} \quad (1)$$

Where, q_e is the electron charge, ϵ_0 is the permittivity of space, and r_{ij} is the distance between particles i and j .

Thomson's problem ranks 7 in the famous list of Steve Smale's [2], 18 problems for our century, which have proved to be useful in many scientific and technological domains ranging from Biology [3] to telecommunication. While Thomson's plum pudding model could not withstand the experimental tests, the irregularities found in the numerical computations of energy values have been found to correspond with electron-shell filling in atoms [4].

Apart from its physical significances, the Thomson's problem is useful in many other fields. In biology, it is the problem of determining the arrangements of the protein subunits which comprise the shells of spherical viruses (e.g. adenoviruses). In structural chemistry, it is the problem of finding regular arrangements for protein's S-layers (e.g. Archaea protein). It also has its applications in other fields as coding theory in mathematics, telecommunications (e.g. the Iridium LEO satellite constellation, cell access points for mobile phones, radio relays locations), sociology, economy, etc.

From an optimization point of view, the Thomson's problem is of great interest to computer researchers also, as it provides an excellent testing problem for new optimization algorithms, due to the exponential growth of the number of configurations with local minimum energy.

Received: April 8th, 2015. Accepted: March 29th, 2016

DOI: 10.15676/ijeei.2016.8.1.3

Recently Min-Yuang Cheng and Doddy Prayogo [5] have developed Symbiotic Organisms Search (SOS), a new and powerful meta heuristic optimization algorithm. This algorithm simulates symbiotic interaction strategies that organisms use to survive in the ecosystem. A main advantage of SOS algorithm over other meta heuristic algorithm is that algorithmic operations do not require any specific algorithmic parameters. This algorithm is more efficient to provide the optimal solutions within very few iterations compared to other heuristic algorithms.

In this paper, mathematical modeling and the various approaches used to solve Thomson's are discussed in section (2). In section (3), an overview of the SOS algorithm used to solve Thomson's problem is presented. The implementation details of proposed algorithm and computational results are elaborated in sections (4) and (5) respectively. The conclusions drawn from the present study are elucidated in section (6).

2. Mathematical formulation of Thomson's problem

Given N identical point charges placed at p_1, p_2, \dots, p_N in the surface of the unit sphere, the electrostatic potential energy between them is given as

$$E = \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{1}{\|p_i - p_j\|} \quad (2)$$

Thomson's problem is to determine the configuration of those points that minimizes the energy in Eq 2. We denote the unit sphere by S^2 in the Euclidean space, such that $S^2 = x^3 \in \mathbb{R}^3: \|x\| = 1$ and $\omega_N = p_1, \dots, p_N$ is the set of the point charges on S^2 . The point $p_i \in S^2$ can be expressed with its three cartesian coordinates, but if we consider a spherical system, $(\hat{e}_{\rho_k}, \hat{e}_{\varphi_k}, \hat{e}_{\theta_k})$, with $k = 1, \dots, N$, omitting the constant sphere radius $\rho = 1$, the point can be parameterized by only two variables $\theta_i \in [0, 2\pi]$ and $\varphi_i \in [0, \pi]$.

Therefore, we have, $p_i, p_j = \hat{e}_{p_i}, \hat{e}_{p_j}$ and $\hat{e}_{p_i} = \sin \varphi_i \cos \theta_i \hat{i} + \sin \varphi_i \sin \theta_i \hat{j} + \cos \varphi_i \hat{k}$, where $(\hat{i}, \hat{j}, \hat{k})$ are the unit vectors in cartesian coordinate system. Therefore, the distance between two point charges is given by: $\|p_i - p_j\| = d_{ij}(\varphi_i, \theta_i, \varphi_j, \theta_j)$.

$$d_{ij}(\varphi_i, \theta_i, \varphi_j, \theta_j) = \sqrt{2(1 - \sin \varphi_i \sin \varphi_j \cos(\theta_i - \theta_j) - \cos \varphi_i \cos \varphi_j)} \quad (3)$$

Where θ_i denotes longitude and φ_i denotes colatitude of the i^{th} point charge, for $i = 1, 2, \dots, N$. Hence our goal is to solve the following minimization problem,

$$\min \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{1}{\sqrt{2(1 - \sin \varphi_i \sin \varphi_j \cos(\theta_i - \theta_j) - \cos \varphi_i \cos \varphi_j)}} \quad (4)$$

With $\theta_i \in [0, 2\pi]$, $\varphi_i \in [0, \pi]$

The objective function is infinitely differentiable and has an exponential number of large local minimum energies. The number of unknowns are $2N$. The fitness value need to be calculated using Equation (4).

Many researchers have tried to solve Thomson's problem, using a variety of methods such as Generalized Simulated Annealing [6], Monte Carlo approaches [7], the steepest-descent and conjugate gradient algorithm [8], [9], Genetic Algorithm [10], non monotone Spectral Gradient Method [11], Particle Swarm Optimization [12], Hybrid Approach using evolutionary algorithm and modified spectral gradient method(EA_SG2) [13]. Thomson's problem serves as an ideal benchmark for testing new global optimization algorithms.

3. Symbiotic Organisms Search (SOS)

The proposed SOS algorithm [5] simulates the interactive behaviour seen among organisms in nature. Organisms rarely live in isolation due to dependence on other species for sustenance and even survival. This dependence-based relationship is known as symbiosis.

A. The basic concepts of symbiosis

The word Symbiosis is derived from a Greek word, means living together. De Bary [5] used the term symbiosis in 1878 to describe the cohabitation behaviour of organisms of different species. Today, symbiosis is used to describe a relationship between any two distinct species. Symbiotic relationships may be either facultative, meaning the two organisms choose to cohabitate in a mutually beneficial but nonessential relationship, or obligate, meaning the two organisms depends on each other for their survival.

The most common symbiotic relationships found in nature are mutualism, commensalism and parasitism. Mutualism is a symbiotic relationship between two different species in which both are benefitted. Commensalism is a symbiotic relationship between two different species in which one benefits and the other is neutral or unaffected. Parasitism is a symbiotic relationship between two different species in which one benefits and the other is badly harmed.



Figure 1. Symbiotic Relationship

Figure 1 illustrates a group of symbiotic organisms living together in an ecosystem. Generally, organisms develop symbiotic relationship as a strategy to adapt to their environmental changes. Symbiotic relationships may also help organisms to increase their fitness and survival reward over the long-term. Therefore, it is reasonable to conclude that symbiosis is present and continues to shape and sustain all modern ecosystems.

B. The Symbiotic Organisms Search (SOS) algorithm

Existing meta heuristic algorithms simulate the natural biological strategy. For example, in Artificial Bee Colony (ABC) [14], [15], it simulates the scrounging behaviour of honeybee swarms. In Particle Swarm Optimization (PSO) [16], it simulates the animal flocking behaviour. In Genetic Algorithm [17], it imitates the process of organic evolution. In SOS algorithm, the process of searching the fittest organism through the symbiotic interactions are used. Initially SOS algorithm was formulated to solve numerical optimization over a continuous search space.

Analogous to other population-based algorithms, SOS algorithm also uses a population of candidate solutions to the necessary areas in the search space in the process of looking for the optimal global solution. SOS algorithm commences with an initial population called the initial ecosystem. In the initial ecosystem, a group of organisms are randomly generated within the search space. Each organism corresponds to one candidate solution of the problem. Each organism in the ecosystem is related to a certain fitness value, that reflects its degree of adaptation to survive.

Almost all metaheuristic algorithms adopts a sequence of operations at each iteration in order to generate new solutions for the next iteration. A standard GA [18] has two parameters, namely crossover and mutation. Harmony Search [19] offers three rules to amend a new

harmony. They are memory considering, pitch adjusting and randomly choosing. Ant Bee Colony algorithm includes three phases to find the best food source. Those phases are the employed bee, onlooker bee, and scout bee phases. In **SOS**, new solution generation is governed by the similar interactions between two organisms in the ecosystem. Three phases that resemble the real biological interaction model are introduced. They are mutualism phase, commensalism phase and parasitism phase.

These characteristics of the symbiotic interaction is the main principle of each phase. The phase of interactions that benefits both the organisms is the mutualism phase; benefits one organism and do not affect the other is the commensalism phase; benefits one organism and actively harm the other is the parasitism phase. Each organism interacts with the other organisms randomly through all phases. This process is repeated for all the organisms in the ecosystems, until termination criteria are met.

Algorithm 1: Symbiotic Organisms Search (SOS) algorithm

```

1 Initialization;
2 for iter = 1 : Maxiter do
3   for i = 1 : ecosize do
4     Mutualism Phase;
5     Commensalism Phase;
6     Parasitism Phase;
7   end
8   if Termination criteria is met then
9     break
10  end
11 end
12 Optimal Solution

```

Algorithm 1 describes these **SOS** algorithm procedures and each procedure is explained in the following section.

B.1 Mutualism phase

Mutualism is a symbiotic relationship, which benefits both participating organisms. For example, the relationship between butterfly and flowers. Butterfly sucks the nectar from the flowers that serves as a source of its livelihood - an activity that benefits butterflies. This activity also benefits flowers as it spreads the pollen grains in the due process, that helps in pollination. This phase imitates such mutualistic relationships.

In **SOS** algorithm, k_i is an organism that corresponds to the i^{th} member of the ecosystem. Another organism k_j is then randomly selected from the ecosystem to relate with k_i . Both organisms involve in a mutualistic relationship to enhance the survival advantage in the ecosystem. New candidate solutions for k_i and k_j are calculated based on the mutualistic symbiosis between organisms k_i and k_j , which is shown in Eq 5 and Eq 6.

$$k_{i_{new}} = k_j + \text{rand}(1, N) * (k_{best} - \text{Mutual_Vector} * BF_1) \quad (5)$$

$$k_{j_{new}} = k_j + \text{rand}(1, N) * (k_{best} - \text{Mutual_Vector} * BF_2) \quad (6)$$

$$\text{Mutual_Vector} = \frac{k_i + k_j}{2} \quad (7)$$

$\text{rand}(1, N)$ in Equation (5) and Equation (6) is a vector of random numbers.

The purpose of BF_1 and BF_2 is interpreted as follows. In nature, some mutualistic relationships might receive a maximum beneficial advantage for one organism than the other. In other words, organism A might receive a large benefit while interacting with organism B. Meanwhile, organism B might only get less benefit while interacting with organism A. Beneficial factors BF_1 and BF_2 are determined randomly as either 1 or 2. These factors represent the degree of benefit to each organisms. Figure 2 shows the different steps involved in the mutualism phase. Equation (7) shows a vector called `Mutual_Vector` that

represents the characteristics of relationship between organism k_i and k_j . The part of equation, $(k_{best} - \text{Mutual_Vector} * BF_2)$, reflects the mutualistic behaviour to achieve the goal of increasing their survival advantage. According to the Darwin's evolution theory [20], only the fittest organisms will prevail, all creatures are forced to increase their degree of adaptation to their ecosystem. Some organisms employ symbiotic relationship with others to enhance their survival habitation. The k_{best} represents the highest degree of adaptation. Hence, k_{best} is the global solution to denote the highest degree of adaptation such that, to increment the fitness of both the organisms. Finally, the new organisms are updated only if their new fitness is better than their previous fitness.

B.2 Commensalism phase

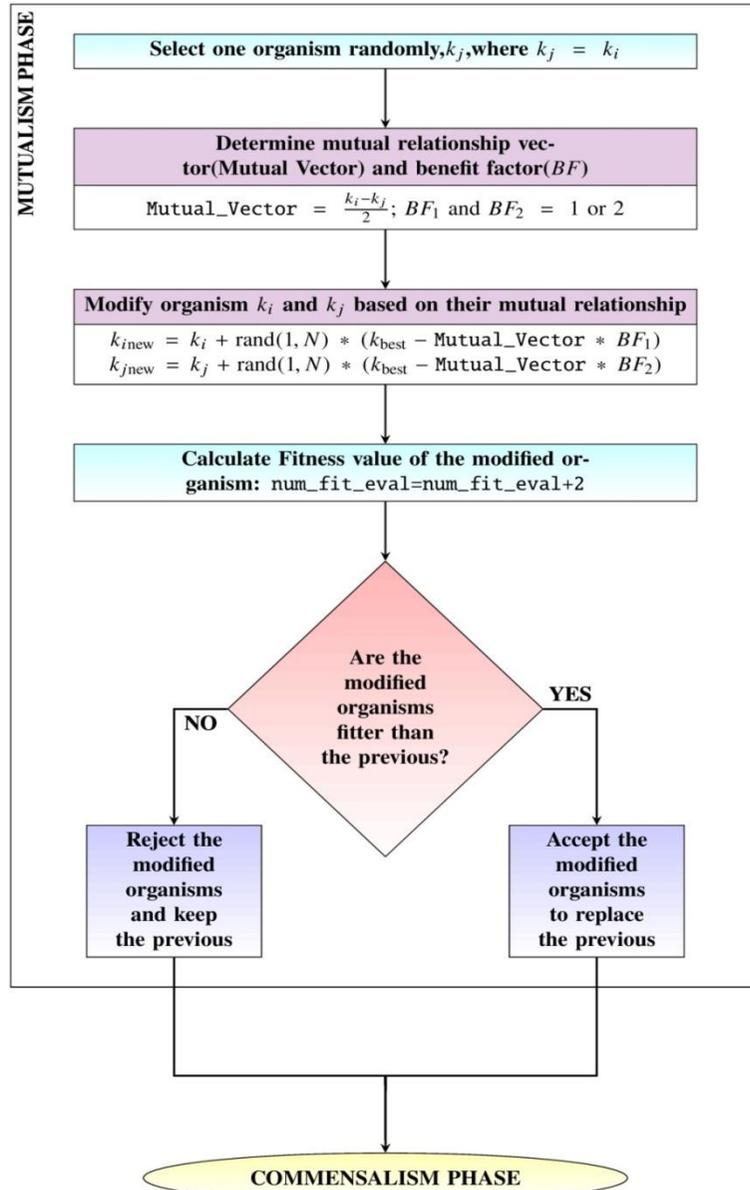


Figure 2. Mutualism Phase

Commensalism is a symbiotic relationship in which one organism is benefitted and the other organism is unaffected. For example, The relationship between remora fishes and sharks. The remora shark sticks firmly to the shark and eats remnant food, thus obtaining a benefit. The shark hardly receives benefit and also unaffected by the activities of remora fish. Figure 3 shows the different steps involved in the Commensalism Phase.

Alike mutualism phase, an organism, k_j is selected randomly from the ecosystem to interact with k_i . In this context, organism k_i tries to benefit from the interaction. However, organism k_j is neither benefitted nor harmed from the relationship. The new solution of k_i is calculated from Equation (8) that depicts a symbiotic commensalism relationship. The organism k_i is updated only if its new fitness is better than its previous fitness.

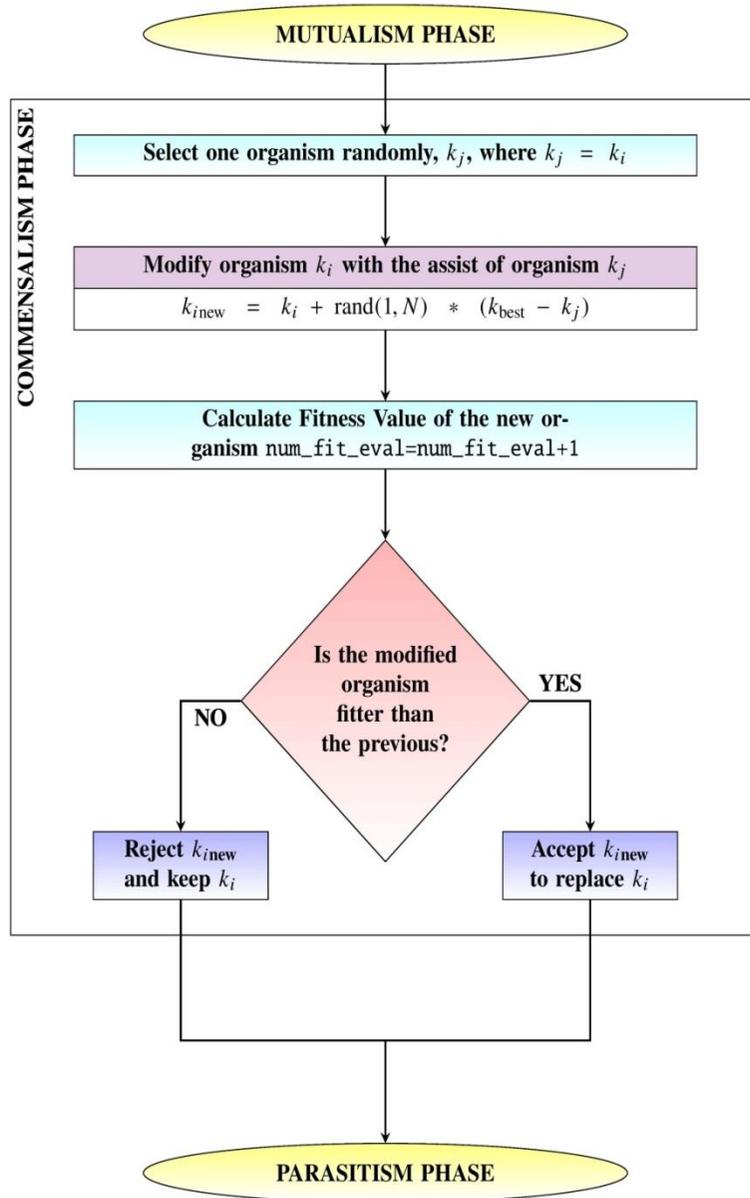


Figure 3. Commensalism Phase

$$k_{new} = k_i + \text{rand}(1, N) * (k_{best} - k_j) \tag{8}$$

In this equation, the expression $(k_{best} - k_j)$, reflects the advantageous benefit provided by k_j to facilitate k_i to enhance its survival in the ecosystem. The highest degree of adaptation in current organism is represented as k_{best} .

B.3 Parasitism phase

Parasitism is a symbiotic relationship in which one organism receives maximum benefit while the other is actively harmed. For example, the symbiotic relationship between plasmodium and human beings. The parasite plasmodium utilizes its relationship with the anopheles mosquito in order to get into the human hosts. The parasite plasmodium flourishes and reproduces inside the human body, thus receiving benefit while the human host suffers malaria which is a fatal disease.

In **SOS** algorithm, organism k_i is given a character similar to the anopheles mosquito in the creation of an artificial parasite called **Parasite_Vector**. **Parasite_Vector** is created in the search space by replicating organism k_i by modifying the randomly selected dimensions using a random number. Organism k_j is selected randomly from the ecosystem to be a host for the **Parasite_Vector**. The **Parasite_Vector** tries to substitute k_j in the ecosystem. The fitness value is compared for both the organisms. If **Parasite_Vector** has a better fitness value, it will kill the organism k_j and occupy its position in the ecosystem. If the fitness value of k_i is better, k_j will be immune to the parasite and k_j will be able to live longer in that ecosystem.

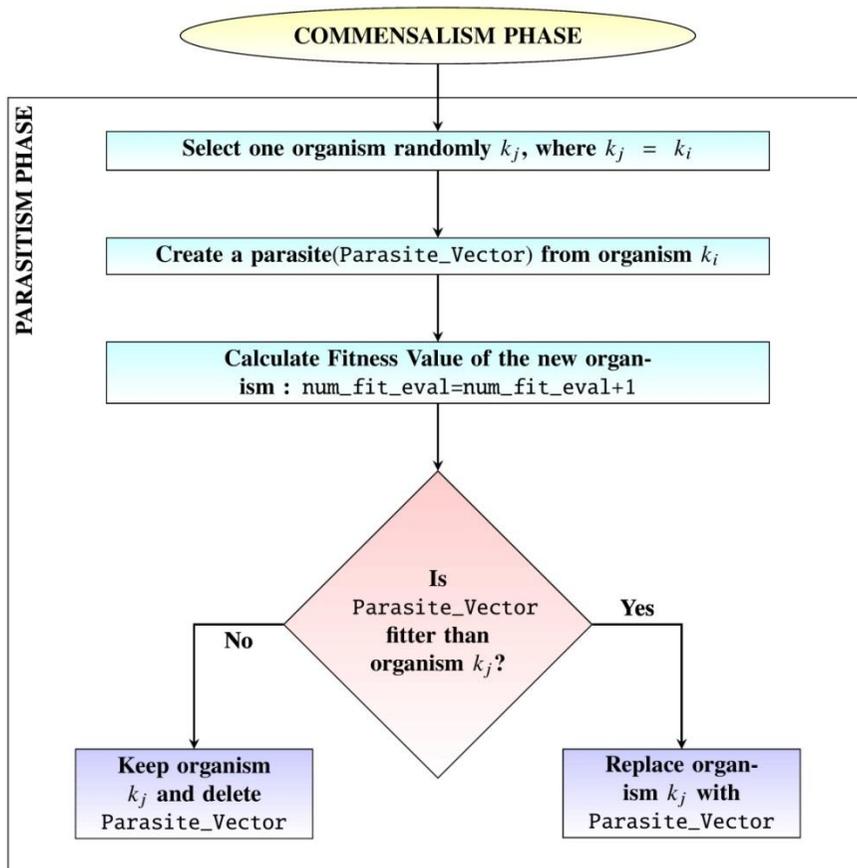


Figure 5. Parasitism Phase

4. Implementation of SOS algorithm for Thomson's problem

This section introduces the step-by-step procedure for implementing SOS algorithm to the Thomson's Problem. The potential energy given in Equation (4) is our objective function. The search space for colatitudes and longitudes are $(0, \pi)$ and $(0, 2\pi)$ respectively. Here the problem is demonstrated for $N = 2$ charges.

Step 1: Ecosystem Initialization

Initially, the ecosize and the maximum number of iterations are set.

- ✓ Number of organisms(eco_size)=10.
- ✓ Maximum iteration(max_iter) =20.

The initial ecosystem is formed by generating random numbers between lower and upper bounds of the search space for the ecosystem size(eco_size) values and a design variable ($N = 2$) number, using random number generator in Matlab. This ecosystem is expressed as,

$$K = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \\ k_7 \\ k_8 \\ k_9 \\ k_{10} \end{pmatrix}, m = \begin{pmatrix} 1.815872 & 0.055946 \\ 0.761127 & 2.870742 \\ 0.392259 & 0.218102 \\ 2.119958 & 1.204165 \\ 0.190531 & 1.652680 \\ 0.345033 & 0.416326 \\ 2.350132 & 0.432687 \\ 2.438811 & 1.905452 \\ 1.997126 & 2.711908 \\ 2.490270 & 0.841787 \end{pmatrix}, n = \begin{pmatrix} 1.744594 & 6.224816 \\ 1.859884 & 2.451022 \\ 3.005663 & 2.931721 \\ 1.765736 & 0.156631 \\ 5.057567 & 1.692916 \\ 0.220873 & 0.509676 \\ 4.572360 & 4.238131 \\ 0.196239 & 5.786889 \\ 3.728094 & 2.142221 \\ 6.206838 & 4.953021 \end{pmatrix},$$

$$X = \begin{pmatrix} 0.631262 \\ 0.568984 \\ 5.706738 \\ 0.640813 \\ 0.628848 \\ 7.806326 \\ 0.607176 \\ 1.337421 \\ 0.891028 \\ 0.600013 \end{pmatrix}$$

where K is the ecosystem, m is the ecosystem for colatitudes, n is the ecosystem for longitudes and X is its corresponding fitness values.

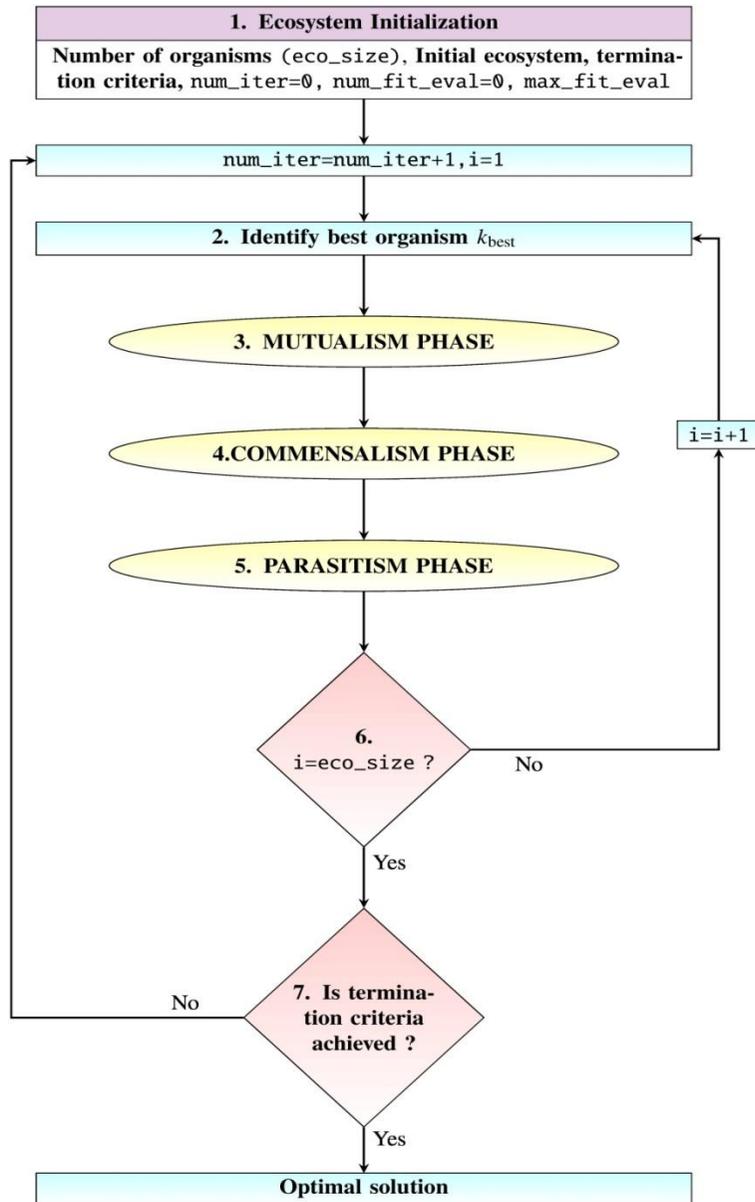


Figure 5. The complete outline of SOS algorithm

Step 2: Identify the best solution

k_2 has the minimum fitness value of the whole ecosystem. In this case k_2 is identified as k_{best} .

Step 3: Mutualism phase

Initially i is set as 1, organism k_1 is matched to k_i . Meanwhile, organism k_j is selected randomly from the ecosystem. In this case, k_5 is selected as k_j . Mutual_Vector is determined using Equation (7). Benefit Factors (BF_1 and BF_2) are determined by randomly assigning value of either 1 or 2.

✓ Mutual_Vector is determined for m and n as (1.0032015, 0.8543130) and (3.4010805, 3.958866) respectively.

- ✓ $BF_1 = 1$ and $BF_2 = 1$. New candidate solutions k_1 and k_2 are calculated using Eq 5 and Eq 6 .

$$k_{1new} = k_1 + \text{rand}(1, N) * (k_2 - \text{Mutual_Vector} * BF_1)$$

$$k_{2new} = k_5 + \text{rand}(1, N) * (k_2 - \text{Mutual_Vector} * BF_2)$$

where k_2 is k_{best} .

m_{1new} and n_{1new} are equal to (0.761127, 2.870742) and (1.859884, 2.451022) respectively with the corresponding fitness value $X = 0.65582$.

- ✓ m_{2new} and n_{2new} are equal to (2.1044, 1.5967) and (5.3377, 3.6164) respectively with the corresponding fitness value $X = 0.66939$. New fitness value of k_{1new} and k_{2new} are compared to the old k_1 and k_2 respectively. Fitter organisms are selected as solutions for the next iteration.
- ✓ If k_{1new} fitness is better than the old, then m_1 and n_1 are modified to m_{1new} and n_{1new} .
- ✓ If k_{2new} fitness is better than the old, then m_2 and n_2 are modified to m_{2new} and n_{2new} .

The flowchart for mutualism phase is shown in the Figure 2 .

Step 4: Commensalism phase

Organism k_7 is selected randomly from the ecosystem. New candidate solutions m_1 and n_1 are calculated using Eq 8.

$$k_{inew} = k_1 + \text{rand}(1, N) * (k_2 - k_7)$$

- ✓ m_{inew} and n_{inew} are (1.9322, 1.3698) and (2.7104, 6.8069) respectively with the corresponding fitness value $X = 0.65582$ are calculated. New candidate solution k_{1new} is compared to the older k_1 . The fitter organism is retained as the solution for the next iteration.
 - ✓ If the fitness value of k_{1new} is $<$ that of k_1 , k_1 is replaced by k_{1new} .
- The flowchart for Commensalism phase is shown in the Figure 3.

Step 5: Parasitism phase

Organism k_{10} is selected randomly from the ecosystem. Parasite_Vector is created by mutating k_i in randomly selected dimensions using a random number within a range between the given lower and upper bounds.

- ✓ Parasite_Vector for m and n are (0.98393, 0.82453) and (2.9814, 1.2017) respectively with the corresponding fitness value $X = 0.81602$. The Parasite_Vector is compared to k_{10} . The fitter organism will survive for the next iteration.
- ✓ The fitness value of Parasite_Vector is better than k_{10} . Therefore, k_{10} is discarded from the ecosystem and replaced by the Parasite_vector.

The flowchart for Parasitism phase is shown in the Figure 4.

Step 6: Go to step 2 if the current k_i is not the last organism of the ecosystem. Otherwise proceed to step 7.

Step 7: Stop if one of the termination criteria is reached; or else return to step 2 and begin the next iteration.

All these seven steps are illustrated in the flow chart in Figure 5. Although Thomson's problem is one of the difficult optimization problems in electrostatics, SOS algorithm determines the optimal solution even before it completes the maximum number of iterations. For $N = 11$ case, the optimal solution reached with just 8 iterations in 0.16s.

5. Computational results

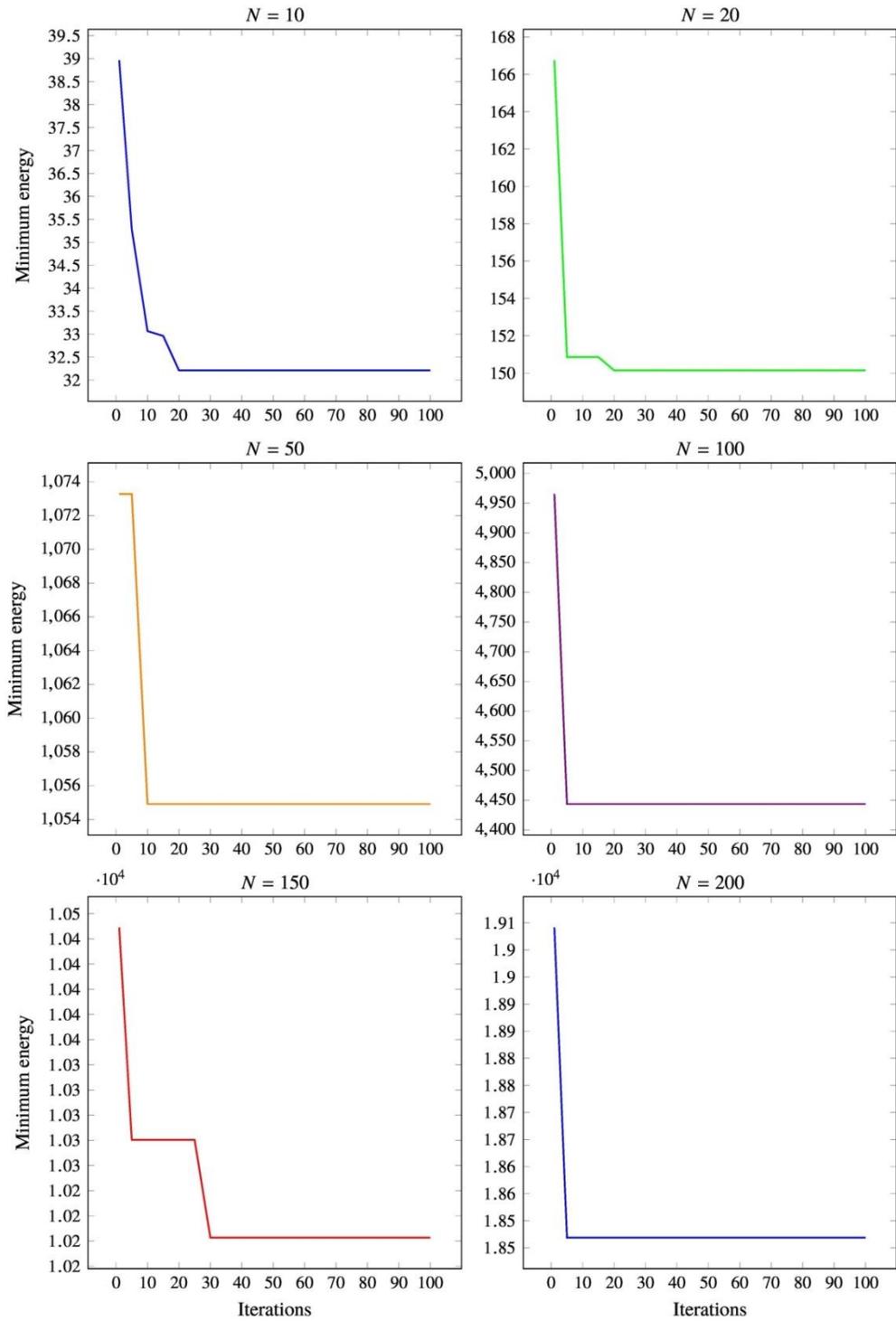


Figure 6. Convergence Plot

Table 1. Comparison of Minimum Energies

Sl.No.	N	EA_SG2 [13]	SOS (proposed)	Run time for SOS (s)
1	3	1.732050808	1.712707	1.882
2	4	3.674234614	3.227460	1.818
3	5	6.474691495	6.422602	1.868
4	6	9.985281374	9.240521	1.866
5	7	14.452977414	14.577197	1.959
6	8	19.675287861	19.856350	1.954
7	9	25.759986531	26.202423	1.962
8	10	32.716949461	32.584820	2.049
9	11	40.596450508	40.249456	2.089
10	12	49.165253058	49.390611	2.264
11	13	58.853230612	58.788443	2.286
12	14	69.306363297	68.008426	2.388
13	15	80.670244114	80.596947	1.089
14	16	92.911655303	92.146954	2.569
15	17	106.050404829	105.049402	2.695
16	18	120.084467447	119.749594	2.871
17	19	135.089467558	134.217825	2.931
18	20	150.881568334	150.053463	3.209
19	21	167.641622400	167.311079	3.343
20	22	185.287536149	184.990704	3.309
21	23	203.930190663	203.840726	3.681
22	24	223.347074052	223.283485	3.588
23	25	243.812760300	243.396099	3.937
24	26	265.133326317	265.399121	4.143
25	27	287.302615033	286.041013	4.432
26	28	310.491542358	310.059125	4.739
27	29	334.634439920	333.691272	4.900
28	30	359.603945904	359.852350	5.123
29	35	498.569872491	497.094549	6.577
30	40	660.675278835	659.794933	8.592
31	45	846.188401061	846.189216	10.507
32	50	1055.182314726	1054.916014	14.320
33	100	-	4443.720433	27.432
34	150	-	10202.694367	106.024
35	200	-	18082.712262	309.761

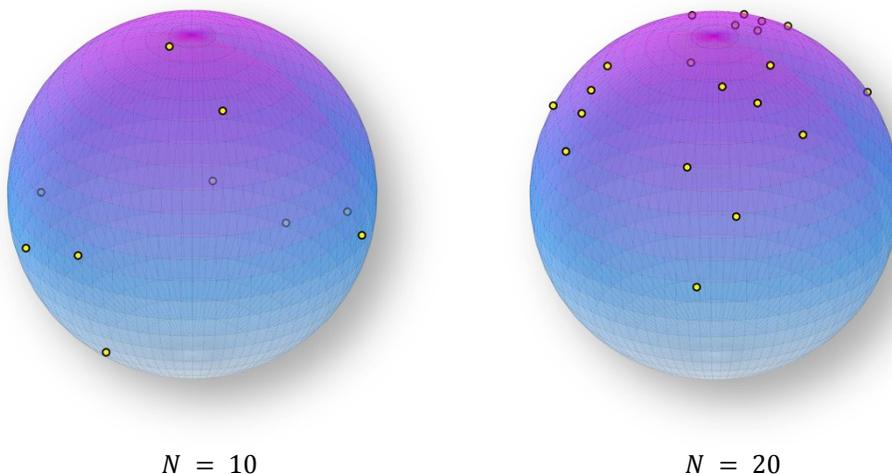


Figure 7. Point charges on a sphere

Table 2. Coordinates values of electrons on unit sphere

N = 10		N = 12		N = 14		N = 16		N = 18	
φ	θ								
0	4.133	0.632	1.093	2.837	1.27	1.229	3.749	3.004	1.674
2.106	0	2.222	0.194	0.756	4.745	0.813	0.556	1.55	1.571
1.581	2.209	0.763	1.732	0.661	0.023	0.846	0.239	1.099	0.42
1.296	2.837	2.869	1.296	1.888	0.757	2.387	0.417	2.183	0.366
2.804	0	2.702	5.005	1.691	1.396	2.13	5.334	2.442	4.163
1.873	1.639	1.251	0.553	1.488	0.85	0.763	2.547	2.586	0.722
0.988	2.558	1.271	3.142	2.482	6.167	1.686	0.778	0.302	0.154
2.448	0	1.735	1.399	1.143	0.684	1.173	2.778	2.023	0.083
0.99	1.48	2.352	3.339	1.996	3.413	1.263	3.108	1.023	4.533
1.873	0.801	1.991	1.944	1.203	1.573	1.288	1.018	2.369	1.456
		1.312	2.238	1.47	0	0.87	2.772	2.396	1.088
		0.86	2.642	0.974	3.038	0.9	0	0.523	4.293
				2.394	1.337	1.661	1.99	1.725	1.558
				1.21	2.716	1.336	2.517	2.258	1.859
						0.533	1.819	2.072	0.377
						1.843	0.392	1.051	2.109
								1.992	1.841
								1.956	2.355

The variable of our problem has the form $(\varphi_1, \varphi_2, \dots, \varphi_N; \theta_1, \theta_2, \dots, \theta_N)$ where N represents the number of points to be distributed on the surface of the sphere. SOS algorithm is implemented using MATLAB R2012a. This algorithm does not require any specific algorithm parameters. The minimum energy values obtained by implementing SOS algorithm is tabulated in the Table 1. From the results presented in Table 1, we can find that in most values of N the SOS algorithm finds good approximate minimum energies given in the literature [13]. The SOS algorithm is also tested for large number of charges (up to a maximum of $N = 200$) and

their corresponding minimum energies are tabulated in the Table 2. The spherical coordinates of the point charges for different values of N are tabulated in the Table 2.

The distribution of ten point charges is tested with different number of iterations and the solutions are tabulated in Table 2. The maximum number of iterations has no effect on the energy values, after reaching convergence. This indicates a good convergence quality of the algorithm. The convergence plot is plotted for different values of N . The program was tested for 100 iterations, but from the Figure 6, we can infer that the optimal solutions converge within very few iterations. The distribution of point charges on a unit sphere is also plotted for $N = 10$ and $N = 20$ and it is shown in the Figure 7.

Table 3. Energies for $N = 10$ for different iterations

Sl.No.	Number of iterations	Energy using SOS	Run Time
1	10	32.116958	0.457
2	20	32.295256	0.851
3	30	32.971663	0.933
4	40	32.911608	1.139
5	50	31.736679	1.340
6	80	32.275188	2.013

6. Conclusion

Symbiotic Organisms Search algorithm is implemented to minimize the energy of point charges on a unit sphere. It is found that SOS algorithm provides better optimum energy values as seen in Table 1. Further, it is noted that SOS algorithm has good convergence characteristics. The convergence time needed for various cases is found to be very minimal, thereby saving computational resources. This method is found to give good approximate values of the energy to obtain optimal configurations of N point charges on unit sphere.

7. References

- [1]. Joseph John Thomson. XXIV. "on the structure of the atom: an investigation of the stability and periods of oscillation of a number of corpuscles arranged at equal intervals around the circumference of a circle; with application of the results to the theory of atomic structure". *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 7(39):23714265, 1904.
- [2]. Steve Smale. "Mathematical problems for the next century". *Mathematical Intelligencer*, 20(2):71415, 1998. doi: 10.1007/bf03025291.
- [3]. Cheng Guan Koay. "Analytically exact spiral scheme for generating uniformly distributed points on the unit sphere". *J Comput Sci.*, 2(1):88 91, March 1 2011. doi: 10.1016/j.jocs.2010.12.003.
- [4]. Tim. LaFave Jr. "Correspondences between the classical electrostatic thomson problem and atomic electronic structure". *J. Electrostatics*, pages 1029 1035, 2013. doi: 10.1016/j.elstat.2013.10.001.
- [5]. Min-Yuan Cheng and Doddy Prayogo. "Symbiotic organisms search: A new metaheuristic optimization algorithm. "Computers & Structures, 139:98 112, 2014. ISSN 0045-7949. doi: <http://dx.doi.org/10.1016/j.compstruc.2014.03.007>. URL <http://www.sciencedirect.com/science/article/pii/S0045794914000881>.
- [6]. Brian Suomela Yang Xiang, Sylvain Gubian and Julia Hoeng. Generalized simulated annealing for global optimization. *The R Journal*, 5, June 1997.
- [7]. L Glasser and A G Every. "Energies and spacings of point charges on a sphere". *J. Phys. A*, 1992. URL (<http://iopscience.iop.org/0305-4470/25/9/020>).
- [8]. T Erber and GM Hockney. "Equilibrium configurations of n equal charges on a sphere". *Journal of Physics A: Mathematical and General*, 24(23):L1369, 1991.
- [9]. T Erber and GM Hockney. "Complex systems: Equilibrium configurations of n equal charges on a sphere". *Advances in Chemical Physics*, Volume 98, pages 49514594, 2007.

- [10]. D. M. Deaven J. R. Morris and K. M. Ho. "Genetic algorithm energy minimization for point charges on a sphere". *Physical Review B*, 1996.
- [11]. Abderrahmane EL Harif Halima Lakhbab, Souad EL Bernoussi. "Energy minimization of point charges on a sphere with a spectral projected gradient method". *International Journal of Scientific & Engineering Research*, 3, March 2012.
- [12]. Elena Bautu Andrei Bautu. "Energy minimization of point charges on a sphere with particle swarms". *Rom. Journ. Phys*, 54(1 2):29 36, 2009.
- [13]. Abderrahmane EL HARIF Halima LAKHBAB, Souad EL BERNOUSSI. "Energy minimization of point charges on a sphere with a hybrid approach". *Applied Mathematical Sciences*, 6(30):1487151495, 2012. URL <http://goo.gl/K7liq6>.
- [14]. Dervis Karaboga and Bahriye Basturk. "A powerful and efficient algorithm for numerical function optimization: artificial bee colony (abc) algorithm". *J Glob Optim*, 39: 45915471, April 2007. doi: 10.1007/s10898-007-9149-x.
- [15]. Dervis Karaboga and Bahriye Basturk. "On the performance of artificial bee colony (abc) algorithm". *Applied soft computing*, 8(1):68715697, 2008.
- [16]. Xuesong Yan, Qinghua Wu, and Hanmin Liu. "A new optimizaiton algorithm for function optimization". In *Advances in Computation and Intelligence*, pages 14415150. Springer, 2009.
- [17]. J.E Beasley and P.C Chu. "A genetic algorithm for the set covering problem". *European Journal of Operational Research*, 94(2):392 404, 1996. ISSN 0377-2217. doi: [http://dx.doi.org/10.1016/0377-2217\(95\)00159-X](http://dx.doi.org/10.1016/0377-2217(95)00159-X). URL <http://www.sciencedirect.com/science/article/pii/037722179500159X>.
- [18]. Lawrence Davis et al. *Handbook of genetic algorithms*, volume 115. Van Nostrand Reinhold New York, 1991.
- [19]. M Mahdavi, Mohammad Fesanghary, and E Damangir. "An improved harmony search algorithm for solving optimization problems". *Applied mathematics and computation*, 188 (2):1567151579, 2007.
- [20]. Mary B Williams. "Deducing the consequences of evolution: a mathematical model". *Journal of Theoretical Biology*, 29(3):34315385, 1970.



G. Kanimozhi obtained her M.Phil in physics from Pondicherry University and presently pursuing Ph.D. in the Department of Physics, Pondicherry Engineering College, Pondicherry, INDIA. Her field of interest are Computational and Theoretical Physics, Optimization Techniques and smart grid networks.



R. Rajathy obtained her B.E in Electrical and Electronics Engineering and M. E in power system with Distinction from Thiagarajar College of Engineering, Madurai and obtained her Ph. D. from Pondicherry University and now working as a faculty in the Department of Electrical and Electronics Engineering, Pondicherry Engineering College, Pondicherry, India. Her field of interest are Power System Optimization, softcomputing techniques, Power System Restructuring and smart grid operations and applications.



Harish Kumar obtained his Ph. D. from Mangalore University, India and now working as a faculty in the Department of Physics, Pondicherry Engineering College, Pondicherry, INDIA. His field of interest are Computational and Theoretical Physics, and Optimization Techniques.